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AN ISOTROPIC THEORY FOR DEWETTABLE SOLIDS

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ABSTRACT:

Fundamental concepts have been formulated for the mechanical behavior of isotropic, elastic materials that dewet. This has been accomplished through an examination of the stability concepts underlying the classical inviscid theory of plasticity. From a phenomenological viewpoint, the two theories differ in the nature of unloading, the dilatation behavior, the form taken by the dewetting criterion, which is analogous to the yield criterion of plasticity theory. Some restrictions to an allowable functional form of the dewetting criterion are developed, and a specific criterion compatible with these restrictions is suggested. A time-temperature superposition method is developed, using the proposed criterion coupled with reaction rate theory. Stress analysis for materials which exhibit anisotropy after dewetting is discussed, and sample problem solutions are given for quasi-elastic pressurization and slow cool-down of an encased hollow cylinder in plane strain, in which the extent of dewetting increases with time.

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PREFACE

This work was supported by the Naval Weapons Center, China Lake, California, which support is gratefully acknowledged. It represents a continuation of work previously reported in NPS-57Li8121A and NPS-57Li981A. A portion of the first section of this report makes a correction of one of the developments in NPS-57Li981A, concerning normality of the dewetted strain increment vector.

This constitutes a progress report outlining the efforts that have produced successful results in the continuing work of building a theory for performing fracture analyses on dewettable materials.

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LIST OF SYMBOLS

ROMAN

e_{ij} = Deviatoric Strain

e = Dilatation Strain

E = Tensile Modulus

f = Dewetting Surface

G = Shear Modulus

I_1 = First Stress Invariant

J_2, J_3 = Deviatoric Stress Invariant

K = Bulk Modulus

S_{ij} = Deviatoric Stress

S = Dilatational Stress

V = Volume

W = Work

A_{ijkl} = Elastic Material Properties

GREEK

α, β = Constants

Δ_{ij} = Residual Strain

∇_{ij} = Deviatoric Residual Strain

∇ = Dilatational Residual Strain

ε_{ij} = Strain

σ_{ij} = Stress

$\sigma_1, \sigma_2, \sigma_3$ = Principal Stresses

SUPERSCRIPTS

d	=	Dewetted Quantity
e	=	Elastic Quantity
o	=	Dewetting Condition
p	=	Plastic Strain Measure
*	=	Initial Loading Condition

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INTRODUCTION

Composite propellants consist of hard, finely ground solid granules embedded in a soft polymeric fuel binder in such a way that each particle is completely encased in a polymeric shell. A chemical bond exists between the polymeric binder and the filler particles, which may constitute as much as 75% to 78% of the volume of the material.

The composite nature of solid propellants provides a capacity for an unusual stress-strain behavior, which is produced by a process of separation of the basic elements of the material. As external loads are applied to the composite, stresses are produced in the bonds between binder and filler. If the stresses are sufficiently large, either the adhesive bonds at the interface are broken, or the cohesive bonds near the interface are broken, and the binder eventually pulls away from the particle leaving a void, or vacuole, surrounding it. This is a dissipative process, and it results in a change in the material properties of the propellant. The phenomenon is called dewetting, and through the manifested change in material properties, it contributes to the nonlinear behavior of the material.

In order to analyze structures fabricated from these materials, it is necessary to develop a mathematical theory of material behavior which

accounts for this nonlinearity. Several idealizations of dewetting are proposed in an effort to gain insight into this type of behavior. A phenomenological approach based upon a continuum is pursued here; wherein the local details of filler particle shape and size, the dissimilarity between binder and filler material, as well as the existence of voids partially or totally surrounding the filler particles is disregarded.

Dewetting is examined in the restricted sense of isothermal, quasielastic behavior; i.e. viscoelastic effects are excluded. The stress-strain law, both loading and unloading, is then further linearized for mathematical simplicity. The simplified behavior is believed to describe many of the essential characteristics of dewetting materials, and from it, information is obtained concerning the dewetted constitutive law and the dewetting criterion.

To accomplish this, the material is examined in light of the stability postulate of Drucker, which has contributed much to plasticity theory. From the postulate several interesting variations from plasticity are developed for dewetting materials; however similarities with this established theory make comparisons with it inevitable. For this reason whenever possible the similarities and differences between the two theories are discussed throughout the development.

A final idealization should be noted. The process of dewetting under virtually all applied loads will produce an anisotropic material. The approach presented herein assumes isotropy, both before and after dewetting, and is intended to be a building block of a more general theory to follow.

NONLINEAR BEHAVIOR IN UNIAXIAL TENSION

A specimen of composite propellant subjected to uniaxial tension typically possesses a nonlinear stress-strain relationship. It is virtually linear up to the onset of dewetting, but at that point the reinforcement of the binder around the particles is reduced, and vacuoles are formed leading to the manifestation of dilatation in the propellant. The next increment of stress produces a correspondingly larger increment of strain due to the reduced modulus resulting from the loss of constraint through dewetting; thus the curve bends over and becomes nonlinear. This process continues as more and more bonds are broken over a greater percentage of the surface of each particle, producing an increase in the volume change and a decrease in the modulus.

Idealization of Stress-Strain Law

From the shape of the curve alone, it is impossible to ascertain precisely the nature of the nonlinear phenomenon being observed. For instance nonlinear elastic or plastic behavior are indistinguishible from dewetting during loading; however, these three are significantly different upon examination of the unloading behavior. Representative forms of actual laboratory traces are given in Figures 1 and 2, which show the real material behavior, including hysteresis. The delineation between the two is quite clear, but such complex histories are difficult to describe mathematically. It has become customary to idealize Figure 1 by the representation shown in Figure 3. Although there is no real material that behaves exactly as depicted, the elements of actual behavior

are included, and the idealization has served a very useful function in engineering analysis. It is proposed that dewetting be idealized as shown in Figure 4, where unloading is elastic with a smaller modulus produced by the breakdown of the material during dewetting.

Within the framework of this idealization, it can be conceived that as a material is loaded in uniaxial tension, it behaves elastically up to the onset of dewetting. If at this point the load is held constant, energy is dissipated within the system by the creation of new surface at the particles. The body, holding the same stress level, now accomodates itself to a new strain level which is incrementally larger than before dewetting. As the load is increased the process is repeated many times as indicated in Figure 5, and in the limit of continuous loading, the stepped curve approaches a continuous curve as dewetting occurs gradually over a range of stress levels. This gives the nonlinear character to the stress-strain curve.

With reference to Figure 5, below the initial dewetting stress, σ^o, the material behaves elastically with modulus E_o. When $\sigma = \sigma^o$ the material is capable of sustaining additional increments of stress with a continuous decrease in modulus. With this model, the change in shape of the stress-strain curve has the general form of propellant behavior. At first the dewetting is slight, and the deviation from E_o is minor, but as dewetting proceeds, the modulus is continuously reduced.

When dewetting is completed at $\sigma = \sigma_D > \sigma^o$, the modulus is once again constant. However, fracture may occur before this happens in many materials, particularly in propellants where the flaw filled binder cannot withstand such high stress levels and follows a non-linear path to failure. At any intermediate point $\sigma^o < \sigma < \sigma_D$, unloading is assumed to occur along a straight line back to the origin. In substance, loading beyond the

linear elastic range has produced a transformation in the microstructure, which in turn has produced a new elastic material with a reduced modulus. After unloading from the nonlinear range, the stress-strain behavior for a second loading is the same as the unloading behavior for the first load application.

DECOMPOSITION OF THE STRAIN

Because of the nonlinear nature of the material in the dewetted region, it is necessary to deal with increments of dewetted strain and subsequently sum them by integration. Such a procedure is classified as an incremental theory for nonlinear materials, and it should be noted that within the assumptions of the quasielastic theory, all nonlinear behavior is assumed to be due to dewetting; i.e., all strains are small enough to be within the realm of linear deformation theory.

In the theory of plasticity it is a fundamental assumption, based on observation, that the total strain is decomposable into elastic and plastic components; i.e.,

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} \tag{1}$$

For dewettable materials, however, this is not the case. In a sense the total strain could be regarded as totally elastic, since it is completely recoverable by an elastic law upon unloading, yet inelastic strains do occur during loading, which contribute to the total strain. In Figure 6, the dewetted increment of strain is decomposed for uniaxial tension. The resulting total incremental contributions and the corresponding portions of strain are uniquely calculable by integration.

To generalize, the conventional assumption of strain decomposition as expressed in equation (1) is replaced in the present development by decomposition of the incremental strain,

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij}^{d} \qquad (2)$$

while using the definition of inelastic increment of strain employed previously by Palmer, Maier and Drucker[1] with reference to a model of elastic-plastic behavior. As adapted here it states:

> The infinitesimal increment of dewetting strain $d\varepsilon_{ij}^{d}$ is the change in strain at σ_{ij}^{o} produced by the application and removal of an infinitesimal stress increment $d\sigma_{ij}$.

CONSTITUTIVE LAW

The dewetted constitutive law is obtainable by applying the above definition of dewetting strain to the proposed model of material behavior. To generalize the one dimensional dewetting behavior to a multiaxial state of stress and strain, it is convenient to introduce a space in which the coordinates of a point are the components of principal stress, σ_i. The dewetting stress in tension is then a point on a dewetting surface $f(\sigma_i^o) = 0$, which is assumed to exist for the type of materials considered here. Generalizing further, there exists about the origin a domain called the elastic range which is defined as the totality of elastic states for which a change $d\sigma_i$ produces no dewetting. If, however, for some $d\sigma_i$, it is possible to produce a nonelastic response, the material is in a dewetted state. The totality of these dewetted states bonds the elastic range and defines the dewetting surface.

To develop the constitutive law, it is convenient to decompose the stress and strain into deviatoric and dilatational components. Any load path in principal stress space is also decomposable into a deviatoric and a dilatational portion. Consider a point on the dewetting surface at S_i^o, which is reached with purely deviatoric loading. (See Figure 7) The stress is related to the strain through the initial elastic constant G_o.

$$S_i^o = 2G_o e_i^o \tag{3}$$

A stress cycle, producing dewetting and returning to S_i^o, creates a dewetting strain increment, de_i^d, which is related to S_i^o through the new modulus G.

$$S_i^o = 2G[e_i^o + de_i^d] \tag{4}$$

The loading increment into the dewetted state, $d\sigma_{ij}$, in general involves both deviatoric and dilatational components. However by the isotropic assumption, the stress and strain components can be decomposed into dilatational and deviatoric parts after dewetting as well as before. By equating (3) and (4) a deviatoric dewetted stress-strain can be formulated

$$de_i^d = \frac{S_i^o}{2}\left[\frac{1}{G} - \frac{1}{G_o}\right] = \frac{S_i^o}{2G_o}\frac{dG}{G} \tag{5}$$

where

$$G = G_o - dG$$

This relationship shows that de_i^d is in the same direction as S_i^o, which in this space is along the line of a vector extending from the origin to the point of intersection of the load path with the dewetting surface. In nine dimensional stress space, the deviatoric constitutive law becomes,

$$de_{ij}^d = \frac{S_{ij}^o}{2G_o}\frac{dG}{G} \tag{6}$$

A similar relationship for a purely dilatational loading to the dewetting surface at S^o, yields

$$de^d = \frac{S^o}{3K_o} \frac{dK}{K} \tag{7}$$

For a general loading, the dewetted strain increment is the appropriate sum of the two basic parts (6) and (7)

$$d\varepsilon^d_{ij} = \frac{S^o_{ij}}{2G_o} \frac{dG}{G} + \frac{S^o}{9K_o} \frac{dK}{K} \delta_{ij} \tag{8}$$

The elastic portion of the strain can also be determined from the cycle. Consider the material just after adding the stress dS_i at S^o_i. A new elastic material has been generated, and the governing relationship between stress and strain at that point is expressed by,

$$S^o_i + dS_i = 2G[e^o_i + de_i] \tag{9}$$

Comparing with equation (4), the nine dimensional form of the relationship is

$$dS_{ij} = 2Gde^e_{ij} \tag{10}$$

The counterpart expression for dilatation is

$$dS = 3Kde^e \tag{11}$$

The complete dewetted constitutive law in incremental form can be written as a sum of (8), (10), and (11)

$$d\varepsilon_{ij} = \frac{dS_{ij}}{2G} + \frac{dS}{9K} \delta_{ij} + \frac{S^o_{ij}}{2G_o} \frac{dG}{G} + \frac{S^o}{9K_o} \frac{dK}{K} \delta_{ij} \tag{12}$$

DRUCKER'S HYPOTHESIS

In 1952 it was shown by Drucker[2] that most of the fundamental results of inviscid plasticity theory were derivable from a single assumption

regarding the positive definiteness of a certain work-like quantity. Drucker[3] extended this concept in 1959 to include inelastic materials with rising stress-strain curves, which he termed stable inelastic materials. The concept upon which Drucker based his work has come to be referred to generally as Drucker's Postulate or Drucker's Hypothesis.

As to the mathematical model of a work-hardening, elastic-plastic material, the hypothesis leads to the expression

$$(\sigma_{ij}^{o} - \sigma_{ij}^{*})\, d\varepsilon_{ij}^{p} + 1/2\, d\sigma_{ij} d\varepsilon_{ij}^{p} \geq 0 \tag{13}$$

where σ_{ij}^{o} is any state of stress lying on the yield surface, σ_{ij}^{*} is any state of stress inside the current yield surface, and $d\varepsilon_{ij}^{p}$ is the increment of plastic strain produced by an outward pointing stress increment $d\sigma_{ij}$ at σ_{ij}^{o}, subject to the assumptions:

1. Continuous variation with plastic strain of shape and position of the yield surface.
2. Continuity and uniqueness (reference) of the stress increment $d\sigma_{ij}$ for an increment of strain $d\varepsilon_{ij}$.
3. Dependence of $d\varepsilon_{ij}^{p}$ upon σ_{ij}^{o} but independence of σ_{ij}^{*}
4. The yield surface is smooth.

There follows from (13) the proofs of

(i) convexity of the yield surface

(ii) Normality of the increment of plastic strain $d\varepsilon_{ij}^{p}$ (or the rate of plastic strain ε_{ij}^{p}) to the yield surface, which leads subsequently to the yielded constitutive law.

Observing the parallels between yield surface and dewetting surface in stress space, between increment of plastic strain $d\varepsilon_{ij}^{p}$ and increment of dewetting strain $d\varepsilon_{ij}^{d}$, and the assumptions associated with their

respective behavior, it is natural to investigate the application of Drucker's Postalate to dewetting materials. Whereas the inequality (13) follows almost by inspection from Drucker's stability postulate, the corresponding expression for the dewetting model requires additional attention due to the fact that the elastic response is altered by the inelastic deformation and is in this sense nonlinear. A feature similar to this in another model (of plastic behavior) has been shown[1] to invalidate the proof of convexity and indeed lead to a nonconvex yield surface.

For a dewetting material whose behavior is described by the idealization shown in Figure 4, and whose initial state of stress referred to above is zero, the postulate is equivalent to the statement that work around a closed cycle is non-zero, or that the work done in dewetting cannot be recovered. For the general case, where the cycle begins and ends at an arbitrary stress level, the postulate has no obvious thermodynamic interpretation. It may be summarized as follows:

> Consider a body at rest acted upon by a set of boundary conditions and body forces. An external agency, distinct from the agency causing the existing state of stress, adds slowly a set of surface tractions and body forces. The displacements of the body will change as the added force is applied and equilibrium maintained. The work done by the external agency must be positive or zero over a cycle of application and removal.

Consequences of Drucker's Postulate for a One-Dimension Stress State

With reference to Figure 6 in a tensile test, suppose after dewetting initiates at A and loading to B that the load is partially removed along BC to σ^*. With point C as a starting point and with Drucker's hypothesis in mind consider the following stress cycle:

(i) Loading from the existing state of stress σ^* at C to $\sigma^o > \sigma^*$ at B, where σ^o is the state at which dewetting is observed to begin.

(ii) Loading from σ^o to $\sigma^1 = \sigma^o + d\sigma$ at D where $d\sigma$ is a positive increment of stress producing an increment of strain $d\varepsilon$ of which a portion is defined as dewetting strain, $d\varepsilon^d$.

(iii) Unloading to σ^* at point E.

It is to be noted here that a stress cycle has been completed in that the body has been returned to the same stress σ^*, but the strain state is different. Dewetting has produced a residual strain $\Delta = \bar{\varepsilon}^* - \varepsilon^*$.

From Figure 6 the work performed by the external agency in the application and removal of stress is represented by the area CBDE which is obviously positive and in agreement with the postulate.

By inspection, the enclosed area is

$$W = 1/2(d\varepsilon^d + \Delta)(\sigma^o - \sigma^*) + 1/2\ d\sigma d\varepsilon^d \geq 0 \tag{14}$$

Since this expression can be shown to be positive, it is to be noted that Drucker's Hypothesis is consistent with the uniaxial behavior. It is a three dimensional expression of this type that is needed to obtain fundamental information about the foundations of dewetting theory.

Drucker's Postulate Applied to Multiaxial Stress States

Consider a state of stress σ^*_{ij} in a region R, where dewetting has not occurred, $(f(\sigma^*_{ij}) < 0)$. The work done by the external agency in loading up to the dewetting threshold, $f(\sigma^o_{ij}) = 0$, is given by

$$W_1 = \frac{1}{2}\int_R (\sigma^o_{ij} - \sigma^*_{ij})(\varepsilon^o_{ij} - \varepsilon^*_{ij})dV \tag{15}$$

After loading to this point, the material is allowed to undergo an increment of dewetted strain $d\varepsilon_{ij}$. During the dewetting phase, the work is

$$W_2 = \int_R \left[(\sigma^o_{ij} - \sigma^*_{ij})\ d\varepsilon_{ij} + 1/2\ d\sigma_{ij} d\varepsilon_{ij}\right] dV \tag{16}$$

Upon unloading to the same stress, σ^*_{ij}, the strain is different, $\bar{\varepsilon}^*_{ij}$, and the work recovered is

$$W_3 = -\int_R \left[1/2(\sigma^o_{ij} - \sigma^*_{ij} + d\sigma_{ij})(\varepsilon^o_{ij} - \bar{\varepsilon}^*_{ij} + d\varepsilon_{ij})\right] dV \tag{17}$$

Combining terms, the work for the cycle is $W = \Sigma W_i$, and from Drucker's Postulate $W > 0$. At any intermediate state of unloading, a strain is reached which is different from that obtained during loading. This produces a residual strain denoted by Δ_{ij}, which at σ^*_{ij} is $\Delta_{ij} = \bar{\varepsilon}^*_{ij} - \varepsilon^*_{ij}$. It is also noted that the volume is arbitrary, permitting the work expression to be written as

$$W = 1/2(\sigma^o_{ij} - \sigma^*_{ij})(d\varepsilon_{ij} + \Delta_{ij}) - 1/2(\varepsilon^o_{ij} - \bar{\varepsilon}^*_{ij})d\sigma_{ij} \geq 0 \tag{18}$$

However, from the dewetting model

$$(\varepsilon^o_{ij} + d\varepsilon^d_{ij}) = A_{ijkl}\sigma^o_{kl} \tag{19a}$$

$$\bar{\varepsilon}^*_{ij} = A_{ijkl}\sigma^*_{kl} \tag{19b}$$

$$d\varepsilon^e_{ij} = A_{ijkl}d\sigma_{kl} \tag{19c}$$

Using these relationships in (18) gives

$$(\sigma^o_{ij} - \sigma^*_{ij})(d\varepsilon^d_{ij} + \Delta_{ij}) + d\sigma_{ij}d\varepsilon^d_{ij} \geq 0 \tag{20}$$

This limit checks to plasticity theory wherein $d\varepsilon^{p}_{ij} = d\varepsilon^{d}_{ij} = \Delta_{ij}$ and equation (20) reduces to equation (13).

SHAPE OF DEWETTING SURFACE

Shape of Cross-section Normal to Hydrostatic Axis

Considering loading histories for which $\sigma^{o}_{ij} - \sigma^{*}_{ij} >> d\sigma_{ij}$, the inequality (20) is written

$$(\sigma^{o}_{ij} - \sigma^{*}_{ij})(d\varepsilon^{d}_{ij} + \Delta_{ij}) \geq 0 \tag{21}$$

When cast in terms of deviatoric and dilatational components, (21) becomes

$$(S^{o}_{ij} - S^{*}_{ij})(de^{d}_{ij} + \nabla_{ij}) + (S^{o} - S^{*})\left(\frac{de^{d} + \nabla}{3}\right) > 0 \tag{22}$$

For a purely deviatoric loading path to dewetting, S = constant,

$$(S^{o}_{ij} - S^{*}_{ij})(de^{d}_{i} + \nabla_{ij}) \geq 0 \tag{23}$$

Recall that $\nabla_{ij} = \bar{e}^{*}_{ij} - e^{*}_{ij}$ is a vector in principal strain space lying in the octahedral plane. Since $S^{*}_{ij} = 2G_{o}e^{*}_{ij}$ and $S^{*}_{ij} = 2G\bar{e}^{*}_{ij}$ then

$$\nabla_{ij} = \frac{S^{*}_{ij}}{2}\left[\frac{1}{G} - \frac{1}{G_{o}}\right] = \frac{S^{*}_{ij}}{2G_{o}}\frac{dG}{G} \tag{24}$$

Substituting (24) and (6) into (23), Drucker's Postulate may be expressed in terms of the stress coordinates of the cycle.

$$(S^{o}_{ij} - S^{*}_{ij})(S^{o}_{ij} + S^{*}_{ij})\ \frac{1}{2G_{o}}\frac{dG}{G} \geq 0 \tag{25}$$

Since $\frac{1}{2G_{o}}\frac{dG}{G} > 0$, equation (25) reduces to

$$S^{o}_{ij}S^{o}_{ij} - S^{*}_{ij}S^{*}_{ij} \geq 0 \tag{26}$$

In stress space, S_{ij}^{o} and S_{ij}^{*} are vectors, and each term in (26) is the dot product of the vector with itself. Equation (26) states that the magnitude of S_{ij}^{o} is always greater than or equal to the magnitude of S_{ij}^{*}. The only way that this can be satisfied is for the contour of the cross-section of the dewetting criterion surface in the octahedral plane to be circular. Thus the shape of the dewetting criterion must exhibit axial symmetry about the hydrostatic axis.

Shape of Cross-section in Plane of Hydrostatic Axis

For a purely dilatational loading path to dewetting, $S_{ij}^{*} = S_{ij}^{o}$ and equation (22) yields

$$(S^{o} - S^{*})(de^{d} + \nabla) \geq 0 \tag{27}$$

From (7) $de^{d} = \dfrac{S^{o}}{3K_{o}} \dfrac{dK}{K}$

When S^{o} and S^{*} lie in the same quadrant,

$$\nabla = de^{d} \frac{S^{*}}{S_{o}} = \frac{S^{*} dK}{3K_{o}K} \tag{28}$$

This relationship does not apply when S^{*} and S^{o} lie in different quadrants, since the bulk modulus after dewetting may take on different values in tension and compression.

With this restriction, substitution of (7) and (28) into (27) yields

$$(S^{o2} - S^{*2}) \frac{dK}{3K_{o}K} \geq 0 \tag{29}$$

First the sign of dK must be determined. If dK is allowed to take on a negative value, then (29) requires $|S^{o}| \leq |S^{*}|$. Since S^{*} is any dilational stress which does not produce dewetting, including the value zero,

this requirement could be met only if S^o is identically zero, contrary to experimental evidence. Thus for any loading path for which S^o and S^* are in the same quadrant, it is concluded that dK must be positive. Then (29) can be satisfied only with the condition that the absolute value of the dilatational stress component at dewetting must be greater than the absolute value of the dilatational stress component prior to dewetting. For any purely dilatational loading path, this restriction means that dewetting can be produced only by an increase in the magnitude of the dilatational stress. In each individual quadrant in principal stress space then the dewetting criterion must be a single-valued function of the dilatational component of stress.

Empirical Consideration

Analysis of the dewetting process on a microscopic scale indicates that dewetting occurs as a result of tensile failure of either cohesive or adhesive bonds in the neighborhood of the filler-binder interface. In the absence of tensile stresses around the filler particles, dewetting does not occur. Although it is not feasible to determine the precise stress values, it is reasonable to assume that imposition of hydrostatic pressure alone creates no tensile stresses in the neighborhood of filler particles. Hence, dewetting will not be produced as a result of hydrostatic pressure alone. This observation suggests that any loading path which starts from an undewetted state and proceeds parallel to the hydrostatic axis in the direction of increasing hydrostatic pressure cannot intersect the dewetting surface. Consequently, the dewetting surface must open continually away from the negative hydrostatic

axis, or, in a limiting condition, parallel the axis. Results obtained from dilatation measurements during uniaxial tension tests under superimposed hydrostatic pressure are in agreement with this hypothesis.

Functional Representation of the Dewetting Surface

The restrictions imposed by (26) and (29), together with the empirical consideration discussed above, can be satisfied by a surface with the characteristics shown in Figure 8. Now these characteristics of the shape of the surface will be translated into restrictions on its functional form.

In a completely general functional form, the dewetting criterion may be given in terms of deviatoric and dilatational stress components by

$$f(S_{ij}, S) = 0 \tag{30}$$

An equally general form for isotropic materials, which is useful in the present case, may be written in terms of stress invariants:

$$f(I_1, J_2, J_3) = 0 \tag{31}$$

where $I_1 = \sigma_1 + \sigma_2 + \sigma_3$ is the first principal stress invariant, $J_2 = -(S_1S_2 + S_2S_3 + S_3S_1)$ is the second principal deviatoric stress invariant, and $J_3 = S_1S_2S_3$ is the third principal deviatoric stress invariant.

The condition derived previously from Drucker's Postulate that the dewetting surface was symmetric about the hydrostatic axis means that the

surface may be described by the specification of two coordinates - the dilatational stress component, and the radius of the cross-section in the octahedral plane, which is given by the octahedral shearing stress. Since the dilatational stress is given by $S = \bar{I}_1/3$ and the octahedral shearing stress is given by $\tau_{oct}^2 = \frac{2}{3} J_2$, the dewetting surface may be described independently of J_3. This is a significant result.

$$f(I_1,J_2) = 0 \tag{32}$$

Without more experimental data than are presently available for dewetting in multiaxial stress states, a specific form of the function in (32) cannot be definitely established. As a first estimation of a specific dewetting criterion, Lindsey[5] has proposed the simplest form, a linear combination.

$$f(I_1,J_2) = I_1 + \alpha J_2^{1/2} - \beta = 0 \tag{33}$$

α and β are constants to be determined from experiment and J_2 is taken to the one-half power for dimensional consistency. In terms of the experimentally determined stress at dewetting in uniaxial tension σ_u, and in equal triaxial tension σ_t, α and β may be evaluated to give

$$I_1 + \sqrt{3}\left(\frac{3\sigma_t - \sigma_u}{\sigma_u}\right) J_2^{1/2} - 3\sigma_t = 0 \tag{34}$$

The dewetting criterion given by (34) describes a surface in principal stress space which is a right circular cone whose axis is the hydrostatic axis and whose vertex is located at $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_t$. The cone opens continually away from the negative hydrostatic axis. This criterion satisfies all the restrictions derived and hypothesized above. At this time no claim can be made that (34) is indeed the correct dewetting criterion; however it can represent real behavior sufficiently well to be useful in analysis. Specific applications are developed in the following sections.

TIME AND TEMPERATURE EFFECTS

In the elastic concept of dewetting behavior discussed in the preceding section, dewetting is considered to occur instantaneously when a critical value of stress is reached. With this concept, if the material is loaded to produce a stress state less than the critical value, dewetting should never occur. In actual experience, however, dewetting does not occur instantaneously upon the application of a suitable load. The time at which dewetting occurs is found to be dependent upon loading rate and temperature, reflecting, in part, viscoelastic material response. However, experimental evidence indicates that temperature and loading rate influence the time to dewetting even for loading conditions normally considered within the range of quasi-elastic behavior. The data reported by Fishman and Rinde[6] for uniaxial tension tests at strain rates less than 1% per minute show such an influence, which incidentally is not correlated by use of the WLF equation[9] normally used for time-temperature superposition of viscoelastic material response characteristics.

For further generalization of the concept of a criterion for dewetting, the influence of time and temperature on the dewetting process will be considered, while retaining the assumption of quasi-elastic stress-strain response. The aim of this effort will be to represent the effects which are peculiar to the dewetting process separately from viscoelastic effects which may also be present in a real material.

In considering the dewetting surface as a function of time and temperature as well as stress, it will first be assumed that the functional form does not change with the introduction of these new variables. The

dewetting surface developed earlier may then be generalized to represent the locus of all stress states for which dewetting will occur at the same time, or at the same loading rate, and at the same temperature. Lindsey's stress criterion for dewetting, eq. (34), may be rearranged in the form

$$\frac{I_1}{3} + \frac{\sqrt{3}}{3}\left(3\frac{\sigma_t}{\sigma_u} - 1\right)J_2^{1/2} = \sigma_t \tag{34a}$$

This has the form in principal stress space of a cone centered on the hydrostatic axis with vertex at $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_t$, the dewetting stress in hydrostatic tension. If dewetting is observed for hydrostatic tensile stress σ_t at a particular temperature and loading rate or time after imposition of load, then the generalized dewetting criterion predicts that dewetting will occur at that same time for any other state of stress represented on the cone with vertex at σ_t, when the temperature and type of loading are the same. If σ_t is a function of time or temperature, the resulting dewetting surface will be displaced along the hydrostatic axis. For example, with the vertex at the origin (zero hydrostatic tension), the time until the occurrence of dewetting is infinite. As the hydrostatic tension is increased, the vertex of the cone moves in the positive direction along the hydrostatic axis, so that instantaneous dewetting would require an infinite hydrostatic tension. Variation with time or temperature of the ratio σ_t/σ_u, on the other hand, would change the vertex angle of the conical dewetting surface.

In order to obtain an analytical expression relating time, temperature, and stress in the dewetting process, the nature of the governing molecular phenomena should be considered. For the purpose of modelling

response of high polymers, the molecular processes involved in deformation under stress are usually divided into three classes:

a. Slippage of chain segments which are relatively free to move about within the polymer network - a purely viscous response.

b. Extension of coiled and entangled chains - a viscoelastic response which may be modeled by multiple Voigt-Kelvin models in series.

c. Rupture of chains through breakage of chemical bonds.

Tobolsky and Eyring[7] presented a mathematical formulation of these responses which they applied to a number of types of deformation processes such as creep, stress relaxation, vibration and rupture in polymeric materials. They state that the viscosity associated with the sloppage of chain segments (a) is much less than the resistance to deformation of processes (b) and (c). They also find that the stresses required for bond breakage are significantly higher than for chain extension and thus assume that for processes of creep and relaxation, except possibly at large deformations or long times, chain extension is the significant molecular process occurring. Bueche[8] has shown that consideration of the response due to chain elongation for different times and temperatures leads to the WLF equation[9] which has been found to represent time-temperature superposition for creep and relaxation data in many polymer systems.

Tobolsky and Eyring published an equation giving the rate of breaking bonds in a polymeric thread under any tensile loading:

$$-\frac{1}{N}\frac{dN}{dt} = \frac{2KT}{h}\exp\left(\frac{-\Delta F}{RT}\right)\sinh\left(\frac{f_o\lambda}{2NKT}\right) \tag{35}$$

f_o = applied tensile stress

λ = distance between equilibrium positions

N = number of unbroken bonds per unit cross-section area

ΔF = apparent energy of activation for the bond breaking process

K, h, R = Boltzman's constant, Planck's constant, and the universal gas constant, respectively.

When the argument of the hyperbolic sine is sufficiently large, as it is under applied loads of the order of 10 psi, and stress and temperature are held constant, (35) may be integrated to obtain the time for which the number of remaining bonds per unit area goes from the initial value N_o, to zero. This is interpreted as the time to failure.

$$t_f = \frac{2N_o h}{f_o \lambda} \exp\left(\frac{\Delta F}{RT} - \frac{\lambda f_o}{2KTN_o}\right) \tag{36}$$

Validity of equations (35) and (36) for the prediction of bond failures is not restricted to polymeric materials. Equation (36) has been found to predict accurately the failure times for uniaxial tensile tests of a great variety of materials. Zhurkov[10], for example, found that an expression of this form predicted fracture behavior for fifty different materials, including metals and alloys, non-metallic crystals and polymers. In an extension to previous work, Graham and Robinson[11] found that failure times for specimens under a time-varying loading history correlated well with predictions obtained by numerical integration of equation (35) for such loading paths as constant strain rate, bi-linear strain rate, and multiple cycles.

The exact molecular behavior involved in the dewetting process is not completely clear. Dewetting may occur in either of two modes: an adhesive failure, in which the bond between filler and binder is broken; or cohesive failure, in which microscopic vacuoles form completely within the binder phase and coelesce to form larger vacuoles at the surface of the filler particles. Mechanisms of viscous flow may contribute to the

formation of the microscopic vacuoles, but the coelescence of vacuoles and the adhesive failure seem to depend significantly on bond breakage. To the extent that this is true, an equation predicting the rate of bond breakage such as the Tobolsky-Eyring equation should be useful for predicting dewetting. One successful application of the Tobolsky-Eyring equation to the prediction of time of dewetting initiation was reported by Graham and Robinson[11] for a polyvinyl chloride plastisol propellant formulation.

The analysis used by Graham and Robinson for their PVC data considered the existence of only one type of failure reaction. They made the further simplifying assumption that the ratio λ/N_o which appears in the reaction rate equation is a linear function of absolute temperature, an assumption on which opinion of various investigators is divided. In the relatively restricted range of temperatures and stresses in which their data were obtained, their representation of the reaction rate theory led to good correlation. However, their correlation was not as close for failure times in a PBAN propellant formulation, especially at low temperatures and high stress rates.

In order to apply reaction rate theory to the dewetting process over the entire range of temperatures and loading rates which may be of interest, it may be necessary to consider more than one type of molecular reaction. For example, it is possible that some propellant compositions may initiate dewetting by a cohesive failure mechanism at high temperatures and an adhesive failure mechanism at lower temperatures or higher loading rates, with different values of reaction rate parameters associated with each process. Also, for loading conditions out of the range of quasi-elastic material behavior, the stress in the reaction rate equation should

reflect viscoelastic response. One method of including visco-elastic response at low temperatures has been proposed by Graham, Henderson and Robinson[12]. Finally, temperature dependence of the reaction rate parameters should also be experimentally determined. The writer has determined that an assumed dependence of the ratio λ/N_o on the square of the absolute temperature yielded a slightly improved fit of the Graham - Robinson PBAN failure data and correlation with failure data for a PBAA formulation reported by Leeming[13]. In any case, the successful correlations obtained by Graham and Robinson indicate that reaction rate theory can well represent the relationship between time, temperature, and stress in the dewetting process.

In order to apply reaction rate theory to the multi-axial dewetting criterion concept, it is necessary to develop a reaction rate equation which reflects the effect of the multi-axial stress state on the dewetting process. This may be done in the following manner.

$$\frac{\sigma_u}{\sigma_t}\frac{I_1}{3} + \frac{\sqrt{3}}{3}\left(3 - \frac{\sigma_u}{\sigma_t}\right) J_2^{1/2} = \sigma_u(T, t_d) \qquad (34b)$$

where T is the temperature and t_d the time at which dewetting occurs. In this expression σ_u and σ_t will be chosen to represent the stresses for which dewetting occurs at the same time and temperature in uniaxial tension and in hydrostatic tension. Consequently, the dewetting criterion will represent the locus of all stress states which will dewet at the same time for which σ_t and σ_u were obtained, when the material is at the same temperature. It is reasonable to suppose that the same molecular reactions, namely bond breakage, govern dewetting in multiaxial stress states as in the uniaxial case. Hence, reaction rate theory would predict that the ratio σ_u/σ_t would be independent of rate or temperature, although each individual term would not. This may be checked experimentally.

Let the left side of equation (34b) define an equivalent uniaxial tensile stress σ_{eq}, This equivalent stress represents the influence of deviatoric and dilatational stress components in the proportions required by the dewetting criterion. In the uniaxial tension case, of course, at the onset of dewetting, $\sigma_{eq} = \sigma_u(T,t_d)$ and σ_u is defined implicitly by the reaction rate equation (35). When, for stress states other than uniaxial tension, the equivalent uniaxial stress

$$\sigma_{eq} = \frac{\sigma_u}{\sigma_t}\frac{I_1}{3} + \frac{\sqrt{3}}{3}\left(3 - \frac{\sigma_u}{\sigma_t}\right)J_2^{1/2} \qquad (37)$$

is substituted in the reaction rate equation, the shape of the dewetting criterion surface will be preserved, as discussed earlier. With this substitution, the reaction rate equation can reflect the effects of time and temperature on the process of dewetting in multiaxial states of stress.

If the Tobolsky-Eyring reaction rate equation is chosen as the governing equation, then, the criterion for dewetting in multiaxial stress states as a function of temperature and time to dewetting is obtained by integration of the equation

$$-\frac{1}{N}\frac{dN}{dt} = \frac{2KT}{h}\exp\left(\frac{-\Delta F}{RT}\right)\ \sinh\left(\frac{\sigma_{eq}\lambda}{2NKT}\right) \qquad (35a)$$

to obtain the time required for the number of unbroken bonds to go from the initial value N_o to zero. In the present development for quasi-elastic material behavior, σ_{eq} may be obtained from an elastic analysis. If the functional form of the stress dependence of the dewetting criterion is the same when viscoelastic response is allowed, then the dewetting criterion could be extended to viscoelastic materials by substitution of the equivalent stress obtained from a viscoelastic solution. When anisotropic response of the dewetted material is allowed, the above criterion must be considered valid only for the initial dewetting.

Movement of Dewetting Surface

For a dewettable material which is in a constant state of stress at a constant temperature, the dewetting criterion in principal stress space is then described by (37). When a particular time to dewetting and temperature are specified, σ_{eq} is the value defined implicitly by (36). The locus of stress states for which σ_{eq} = constant generates a cone about the hydrostatic axis which is also the locus of stress states with the same time to dewetting. Thus the time-dependent dewetting criterion in principal stress space may be described in terms of a moving conical surface. At time zero the cone is located at infinity on the hydrostatic axis. With increasing time, the cone moves along the hydrostatec axis toward the origin. Dewetting is predicted to occur when the existing stress state intersects the cone surface, shown schematically in figure 10.

When stress and temperature are allowed to vary with time, the time to dewetting must be calculated by integration of (35). Thus the rate of motion of the dewetting surface is dependent upon the loading path.

CONSIDERATION OF DEWETTING-INDUCED ANISOTROPY

The development in the preceding section has considered the case of a material which is isotropic and linearly elastic both before and after dewetting. As a first step toward greater generality, the affects of anisotropy in the dewetted phase of the material will be considered, while still restricting response characterization to the elastic or quasi-elastic case.

The initial dewetting criterion is assumed to depend only upon stress, and the material is considered to remain isotropic until dewetting occurs.

Thus the stress criterion for initiation of dewetting in a previously undamaged material may still be given by a function of the form of eq. (34). However, the criterion for initial dewetting will be affected to some degree by the type of material response which follows dewetting. As White and Drucker[14] have discussed in connection with plasticity theory, when anisotropy is introduced as a result of deformation, the subsequent loading function (in this case, the dewetting criterion) must depend explicitly upon the components of dewetted strain, as well as stress. Consequently, the dewetting function is no longer a path-independent function, but depends upon the prior history of dewetting in the material. Similarly, the dewetted constitutive law is also history-dependent. This may be demonstrated with a simple example. A sheet of dewettable material is stretched in uniaxial tension until dewetting occurs (Fig 11a). The onset of dewetting is predicted by the previously developed criterion. As numerous visualization studies have demonstrated, dewetting results in separation of binder and filler particles in the direction of strain and produces an associated reduction of modulus in that direction. Now if the same sheet is next stretched in a different direction (Fig 11b), its response will depend upon the orientation with respect to the previous direction of dewetting. This new response must be considered in order to predict the point at which further dewetting will occur.

Material Orthotropy

Some information about the nature of anisotropic material response resulting from initial dewetting can be ascertained from a particular model. The composite material is represented as an orderly array of uniform spherical filler particles within a uniform strain field each

surrounded by binder. Figure 12 illustrates a cubical building block of such an array, oriented with sides parallel to the principal strain directions, with dewetting in the direction of ε_1. The shaded areas of the sphere indicate regions in which the filler-binder bond has been broken. Since the faces of the cube are planes of symmetry of the array, they remain plane during deformation with their relative motion given by the values of the principal strains.

As has been observed by Elliott[15] the areas of dewetted surface (shaded areas) and the shapes of the resulting vacuoles are influenced by the relative magnitudes of the three strains. Indeed, if the dewetted area were specified, an elastic solution could be obtained relating the average stresses on the walls of the deformed cube to the strains at the walls, giving, in effect, the anisotropic constitutive law of the material. If the extent and shape of the dewetted area of the spherical inclusion were a known function of the strains which exist when the dewetting criterion is reached, then the dewetted constitutive law for the array could be calculated from a knowledge of the pre-dewetted strain field.

At present such a solution is not available. However, if the assumption is made that the shape and extent of the dewetted area is uniquely determined by the strain field which exists at the initiation of dewetting, some information about the resulting anisotropy may be deduced. With this assumption, the dewetted material will exhibit a geometric anisotropy possessing axes of symmetry paralleling the directions of principal strains which exist prior to dewetting. Since, in theory at least, the dewetted constitutive law is uniquely determined once the geometry is specified, the material anisotropy will also have axes of symmetry corresponding to the directions of principal strain. If a general

curvilinear coordinate system were chosen whose coordinate directions correspond everywhere with principal strain directions, then those coordinate directions would be axes of symmetry of the anisotropic dewetted material.

The particular type of anisotropy which exhibits symmetry about three orthogonal directions is classified as orthotropy. Thus, with reference to a coordinate system corresponding with principal strain directions of the pre-dewetted material, a material after initial dewetting may be classified as orthotropic. In such a coordinate system, the number of independent material moduli required to describe the dewetted material is reduced from thirty-six to nine.

When the strain field is uniform within a pre-dewetted structure, then the resulting anisotropy will be homogeneous. When a nonuniform strain field exists prior to dewetting, then the dewetted material will be non-homogeneous with a spatial variation in properties directly related to the variation in principal strains. For loading geometries which produce complex patterns of principal strains in the isotropic elastic solution, analytical evaluation of the stress-strain solution including dewetted regions will be complicated. However, one problem of engineering interest which permits further simplification and analytical solution is a circular cylinder under axisymmetric loading in plane stress or plane strain conditions. For this particular geometry the directions of principal strain and stress in the isotropic elastic material are parallel with the coordinate directions of a standard cylindrical coordinate system. Consequently, the ensuing dewetted material will be orthotropic with respect to the same coordinate system. Cylindrical orthotropy when reduced to the case of plane stress requires only three independent moduli to characterize response. The constitutive law is given in terms of

engineering moduli by

$$\begin{Bmatrix} \varepsilon_r \\ \\ \varepsilon_\theta \end{Bmatrix} = \begin{bmatrix} 1/E_r & -\nu_{r\theta}/E_\theta \\ \\ -\nu_{\theta r}/E_r & 1/E_\theta \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \\ \sigma_\theta \end{Bmatrix}$$

and symmetry of the coefficient matrix requires $\nu_{r\theta}/E_\theta = \nu_{\theta r}/E_r$. For the case of plane strain, the constitutive law becomes

$$\begin{Bmatrix} \varepsilon_r \\ \\ \varepsilon_\theta \end{Bmatrix} = \begin{bmatrix} 1/E_r(1 - \nu_{zr}\nu rz) & -1/E_\theta(\nu_{r\theta} - \nu_{rz}\nu_{z\theta}) \\ \\ -1/E_r(\nu_{\theta r} - \nu_{zr}\nu_{\theta z}) & 1/E_\theta(1 - \nu_{z\theta}\nu_{\theta z}) \end{bmatrix} \begin{Bmatrix} \sigma_r \\ \\ \sigma_\theta \end{Bmatrix}$$

with a corresponding symmetry requirement. Since there is no angular variation in strain for the case of axial symmetry in load and geometry, the anisotropic moduli will at most be functions of radial position, reflecting the radial variation of principal strains.

Bilinear Approximation

In the general case, the constitutive laws of a composite material both before and after dewetting may be nonlinear functions of strain, reflecting material nonlinearity. For many compositions when loaded quasi-elastically, the uniaxial stress-strain curves may be closely approximated by a bilinear constitutive law, shown schematically in Figure 13. For loading the material is considered to have a constant modulus prior to dewetting and to transition abruptly to a new constant modulus reflecting dewetted behavior when a critical value of strain or stress is reached. A model developed by Farris[16] relating dilatation to the dewetting process shows that this bilinear idealization is equivalent to the assumption that all available filler particles dewet simultaneously when the critical value of strain is reached. In order to simplify the

the following analysis, this bilinear idealization of loading will be employed. Of course unloading will be handled in the manner described previously. Thus, material tangent moduli will be considered to have constant values, both before and after dewetting.

Example Solutions

Two particular problems of engineering interest in the solid rocket field are internal pressurization and slow cooling of the hollow cylinder of composite solid propellant bonded in a flexible case of higher modulus. Conditions of plane strain are assumed. Linear isotropic elastic solutions for each of these examples are obtained by classical means. A convenient tabulation is given in Reference 17. The dewetting criterion, equation (34a), with the dewetting stress in equal triaxial tension σ_t taken as twice the dewetting stress in uniaxial tension σ_u in agreement with Lindsey[5], predicts that dewetting occurs first at the inner bore of the solid propellant for both cases under consideration. Thus, the problem to be solved is a three-element composite circular cylinder, illustrated in Figure 14. The inner ring is composed of dewetted material and is cylindrically orthotropic. The center ring is undewetted propellant and is isotropic. The outer layer is the rocket case, and may be either anisotropic or isotropic but will be considered isotropic in the present analysis.

Solutions have been given by Lekhnitskii[18] for a number of problems involving cylindrically orthotropic structures loaded in plane stress, including one analysis in which material moduli were allowed to vary radially according to a specific power law. In addition, a solution has been published by Freudenthal[19] for stresses due to slow cooling of an

orthotropic cylinder bonded to a rigid case. Using the methods of Lekhnitskii, solutions have been obtained for the geometry shown in Figure 14 for stresses within the composite cylinder as a function of the position of the interface between dewetted and undewetted propellant, for conditions of plane stress and plane strain. The effects of radial variation of the anisotropic moduli are disregarded. Also, in the absense of any information to the contrary, the coefficient of thermal expansion for the composite propellant is assumed to be unaltered by the dewetting process. The resulting formulas are given in Appendix I. These formulas were evaluated with the use of a digital computer for a particular geometry and physical properties estimated to be representative of a typical steel-encased analog rocket motor. The particular values used and results obtained are discussed in Appendix II.

Using the reaction rate equation of Tobolsky and Eyring with the value of equivalent stress defined in (37), the position of the interface between dewetted and undewetted material as a function of time is calculated for the previously discussed problems of internal pressurization and slow cool-down of an encased cylinder in plane strain. For the problem solution, it is assumed that the differential form of the reaction rate equation (35), could be applied for time-dependent values of both stress and temperature. The solution proceeds in the following steps:

1. The isotropic linear elastic solution is examined to determine the position at which the highest value of equivalent stress is located. In all cases this is the inner boundary.
2. The reaction rate equation is integrated to determine the time at which material at one radial increment from the inner boundary dewets. In the case of internal pressurization, a constant stress level is used. For the cool-down problem, a constant rate of

temperature decrease is considered, and the associated stress history is used.

3. After dewetting takes place at the inner boundary, a new stress solution is calculated for a three-element cylinder with the inner anisotropic ring one radial increment thick.

4. Next, the time to dewetting is calculated for a radial position two increments out from the inner boundary. For the case of internal pressurization, the stress history at this point is:

   a. constant stress as given by the isotropic solution from time zero until the calculated time at which the first increment dewets.

   b. a second value of constant stress, as given from the three-element anisotropic solution. A similar step change in stress history is calculated for the cool-down problem.

5. The point of calculation again moves by one radial increment and the time at which the dewetting interface reaches that point is calculated as in step 4.

6. The process is continued until the entire cylinder dewets or until the equivalent stress in the undewetted area reaches zero.

Calculations are performed with the aid of a digital computer program. No computing problems have been encountered to date. The longest computer running time experienced so far have been under one minute. A discussion of the specific data used in calculation and the results obtained is listed in Appendix III.

SUMMARY

The present development constitutes a first step toward a theory of structural analysis of dewettable materials loaded to produce multiaxial states of stress. Idealization of the material as isotropic and elastic, both before and after dewetting, permits consideration of the nonlinear variation of material properties associated with dewetting independently from other nonlinear effects associated with most dewettable materials. Treatment, analogous in many respects to plasticity theory, yields information about a stress criterion for the initiation of dewetting. It is found that the criterion for dewetting is independent of the third deviatoric stress invariant, and thus may be described in terms of two independent variables: the nean hydrostatic stress, and the octahedral shear stress; or, alternatively, the invariants I_1 and J_2. For illustrative purposes, the criterion $\alpha I_1 + J_2^{1/2} - \beta = 0$ satisfies the constraints of the development, and is in agreement with the limited amount of available information concerning dewetting of real materials under quasi-elastic loading.

Effects of time and temperature on the process of dewetting in an otherwise elastic material can be considered with the use of a reaction rate theory for molecular bond breakage. The dewetting criterion provides an equivalent bond stress for generalization of the essentially uniaxial reaction rate theory to multiaxial stress states.

A method of analysis which includes the effects of anisotropy induced by dewetting according to the above criterion is developed. It

is concluded that upon first dewetting, a material assumes the characteristics of orthotropy with reference to a general curvilinear coordinate system which coincides with the directions of the principal stresses in the pre-dewetted state. Stress solutions in closed form are given for the specific problem of an encased, hollow, circular cylinder of dewettable material loaded in plane strain by either internal pressurization or slow cooling. With these stress solutions, the time histroy of dewetting within the cylinder is calculated by integration of a reaction rate equation.

Experimental investigation is necessary to justify the choice of any specific form of dewetting criterion, to determine the values of the parameters used in the reaction rate equation and to evaluate the anisotropic moduli of the dewetted material. When these values are obtained, the method of analysis discussed herein should provide meaningful quantitative results within the limitations imposed by the quasi-elastic idealization. Alternatively, qualitative bounds on structural response may be obtained by parametric studies.

The approach used in the development of an analysis for the idealized material considered in this report may serve as a basis for a more general analysis which should consider effects of viscoelasticity and anisotropy in greater detail.

REFERENCES

1. A. C. Palmer, G. Maier, and D. C. Drucker, "Normality Relations and Convexity of Yield Surfaces for Unstable Materials or Structural Elements," Journal of Applied Mechanics Vol.34, Trans. ASME Vol 89 Series E, 1967, pp. 464-470.

2. D. C. Drucker, "A More Fundamental Approach to Plastic Stress-Strain Relations," Proceedings of the First U. S. National Congress of Applied Mechanics, New York, N.Y. 1952, p. 487-491.

3. D. C. Ducker, "A Definition of Stable Inelastic Materials," Journal of Applied Mechanics, Vol 26, Trans ASME Vol 81, Series E, 1959, p. 101-106.

4. B. Paul, "Macroscopic Criteria for Flow and Fracture," in Fracture, an Advance Treatise, Vol 2, 1968, p. 333.

5. Lindsey, G. H., "The Effects of Dewetting on Fracture of Solid Propellants," U. S. Naval Postgraduate School Report NPS-57Li8121A, December 1968.

6. Fishman, N. and J. A. Rinde, I.C.R.P.G. Mechanical Behavior Working Group Second Meeting Bulletin, October 1963.

7. Tobolsky, A. and H. Eyring, "Mechanical Properties of Polymeric Materials," Journal of Chemical Physics, v. 11, p. 125-134, March 1943.

8. Bueche, F., Physical Properties of Polymers, Interscience Publishers, 1943.

9. Williams, M. L., R. F. Landel and J. D. Ferry, "The Temperature Dependence of Relaxation Mechanisms in Amorphous Polymers and Other Glass-Forming Liquids," Journal of the American Chemical Society, Vol. 77, p. 3701-3706, July 20, 1955.

10. Zhurkov, S. N., "Kinetic Concept of the Strength of Solids," International Journal of Fracture Mechanics, Vol. 1, No. 4, p. 311-323, December 1965.

11. Robinson, C. N., P. H. Graham, and F. C. Moore, Solid Propellant Mechanical Behavior Studies, Vol. II, Technical Report AFRPL-TR-69-124-Vol. II, May 1969.

12. Graham, P. H., C. B. Henderson, and C. N. Robinson, "Description of Failure Processes by means of an Extended Reaction Rate Model," I.C.R.P.G. Mechanical Behavior Working Group Eighth Meeting Bulletin, November 1969.

13. Leeming, H., "Failure Behavior of Highly Filled Rubber Based Propellants," I.C.R.P.G. Mechanical Behavior Working Group Sixth Meeting Bulletin, December 1967.

14. White, G.N., Jr. and D. C. Durcker, "Effective Stress and Effective Strain in Relation to Stress Theories of Plasticity," Journal of Applied Physics, Vol. 21, p. 1013-1021, October 1950.

15. Elliott, D. M., "An Investigation of the Nature of Microstructural Damage in a Filled Elastomeric Composite Material," Ph.D. Thesis Texas A. & M. University, January 1968.

16. Farris, R. J., "The Influence of Vacuole Formation on the Response and Failure of Filled Elastomers," Transactions of the Society of Rheology, 12:2, p. 315-334, 1968.

17. Williams, M.L., P. J. Blatz and R. A. Schapery, "Fundamental Studies Relating to Systems Analysis of Solid Propellants," GALCIT SM 61-5, California Institute of Technology, February 1961.

18. Lekhnitskii, S. G., Anisotropic Plates, Gordon and Breach, New York, 1968.

19. Freudenthal, A.E., "Temperature and Shrinkage Stresses in Orthotropic Cylinder Bonded to a Rigid Case," AIAA Journal, Vol. 6, No. 2, p. 376-378, February 1968.

20. Francis, E. C. and C. H. Carlton, "Some Aspects of Nonlinear Mechanical Behavior of a Composite Propellant," Journal of Spacecraft, Vol. 6, No. 1, January 1969.

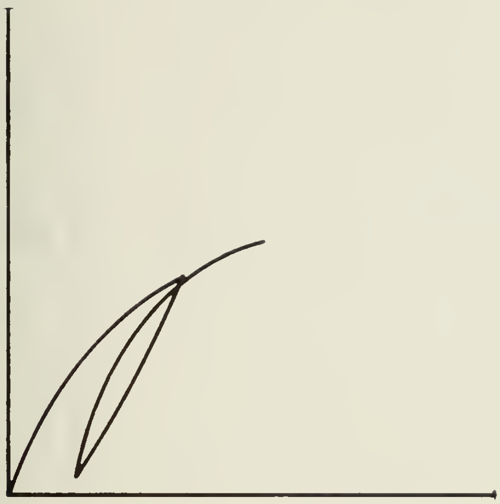


FIGURE 1. PLASTICITY WITH HYSTERESIS

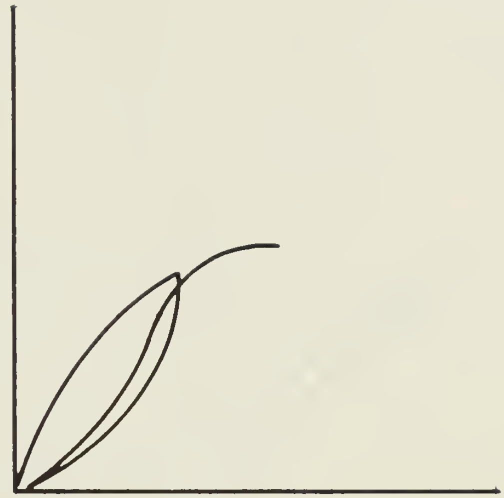


FIGURE 2. DEWETTING WITH HYSTERESIS

FIGURE 3. PLASTICITY MODEL

FIGURE 4. DEWETTING MODEL

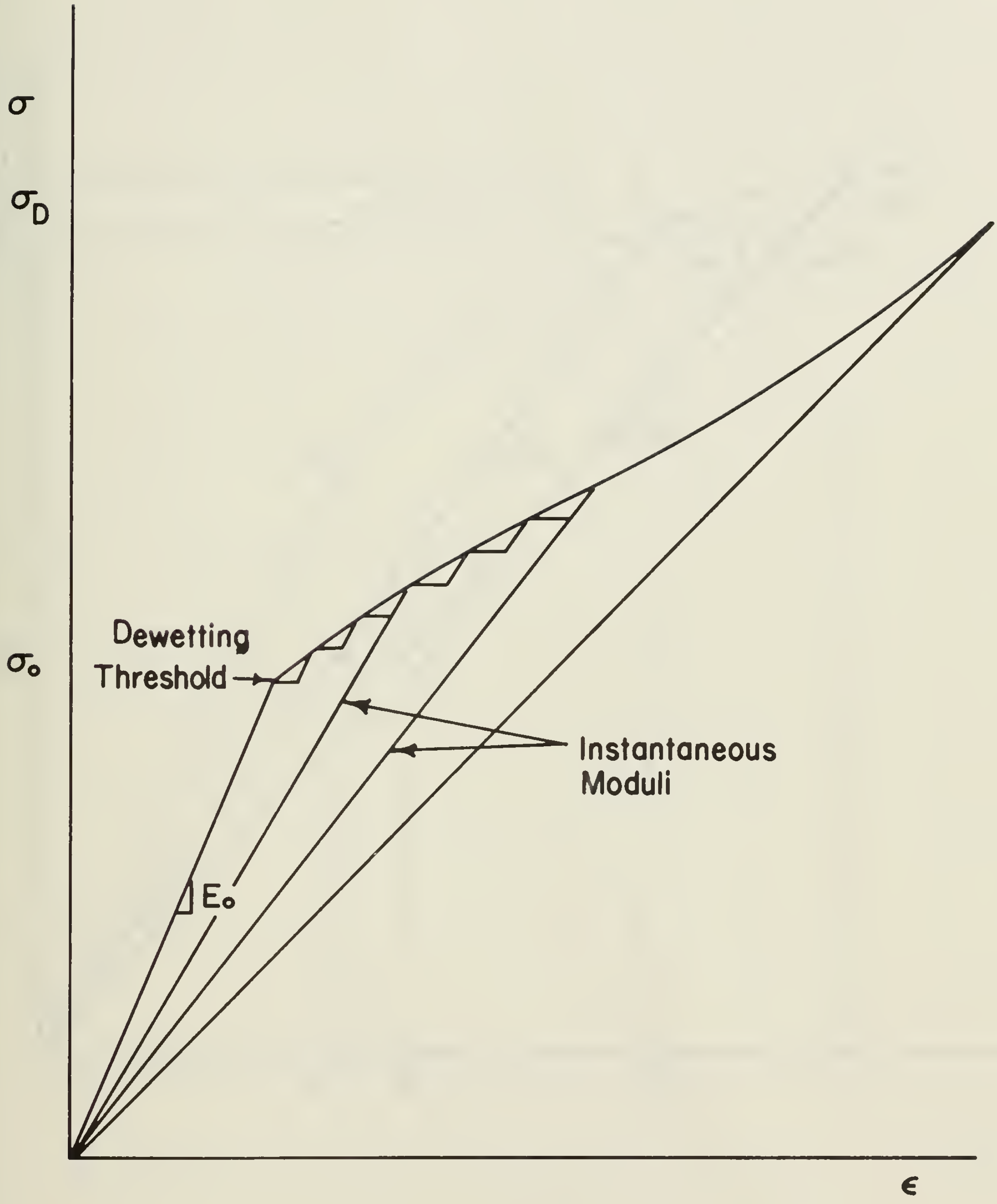


FIGURE 5. IDEALIZATION OF QUASIELASTIC DEWETTING BEHAVIOR

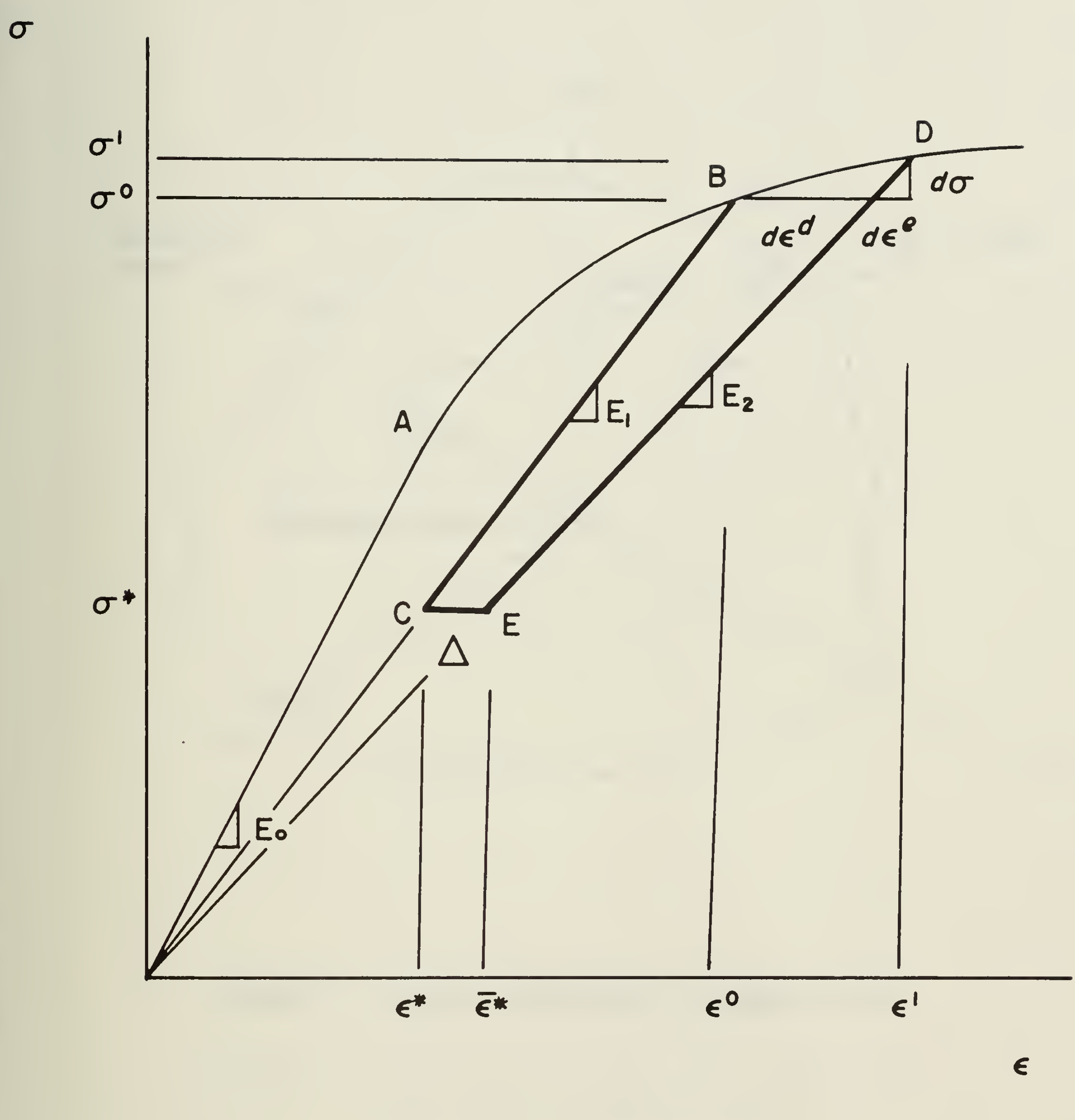


FIGURE 6. STRESS CYCLE FOR A DEWETTING MATERIAL

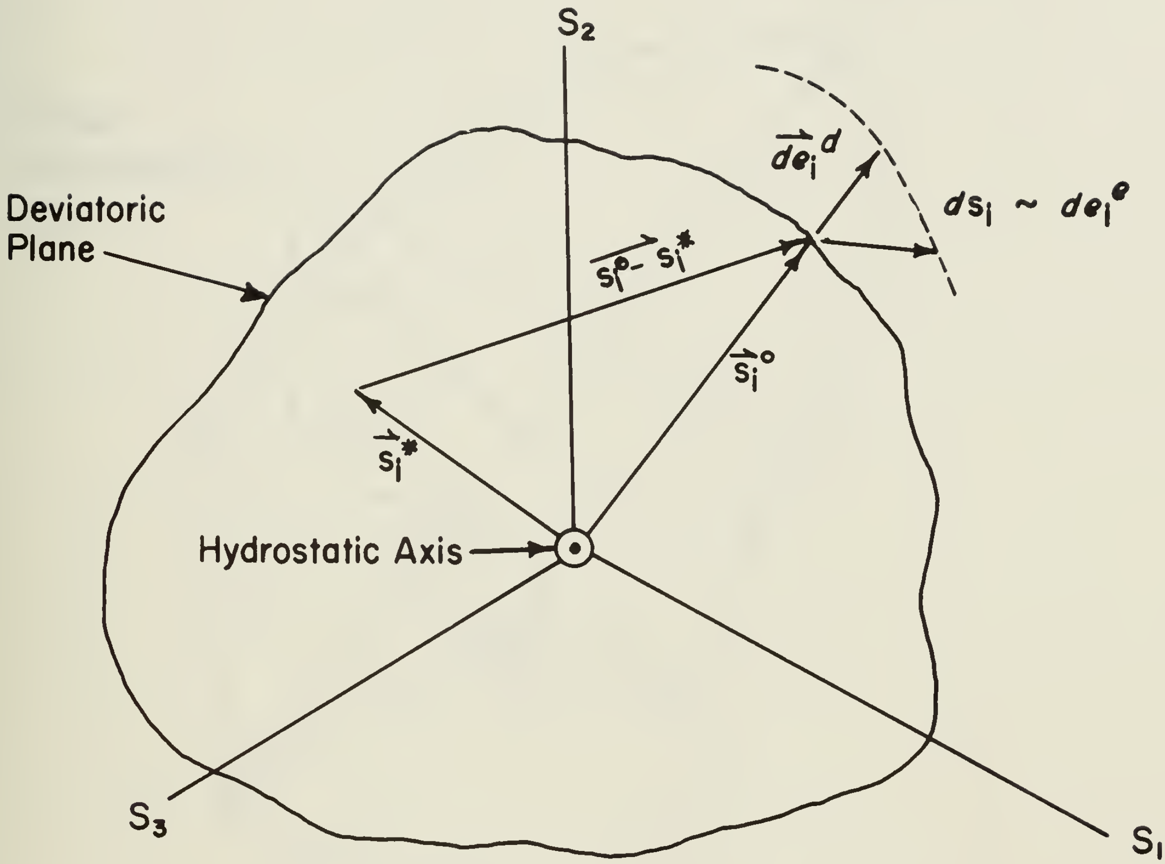


FIGURE 7. PURELY DEVIATORIC LOADING PATH

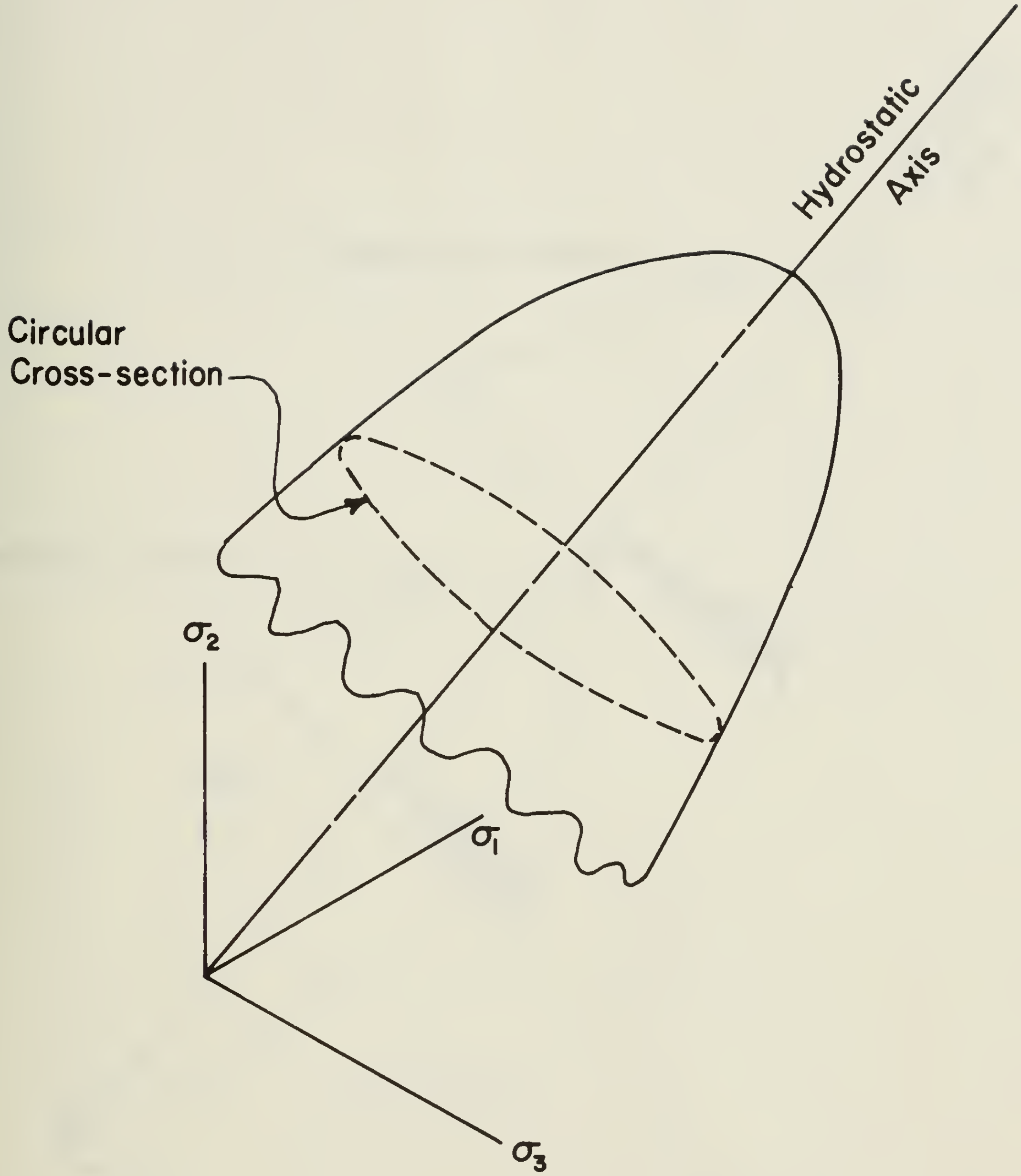


FIGURE 8. ALLOWABLE DEWETTING CRITERION IN PRINCIPAL STRESS SPACE

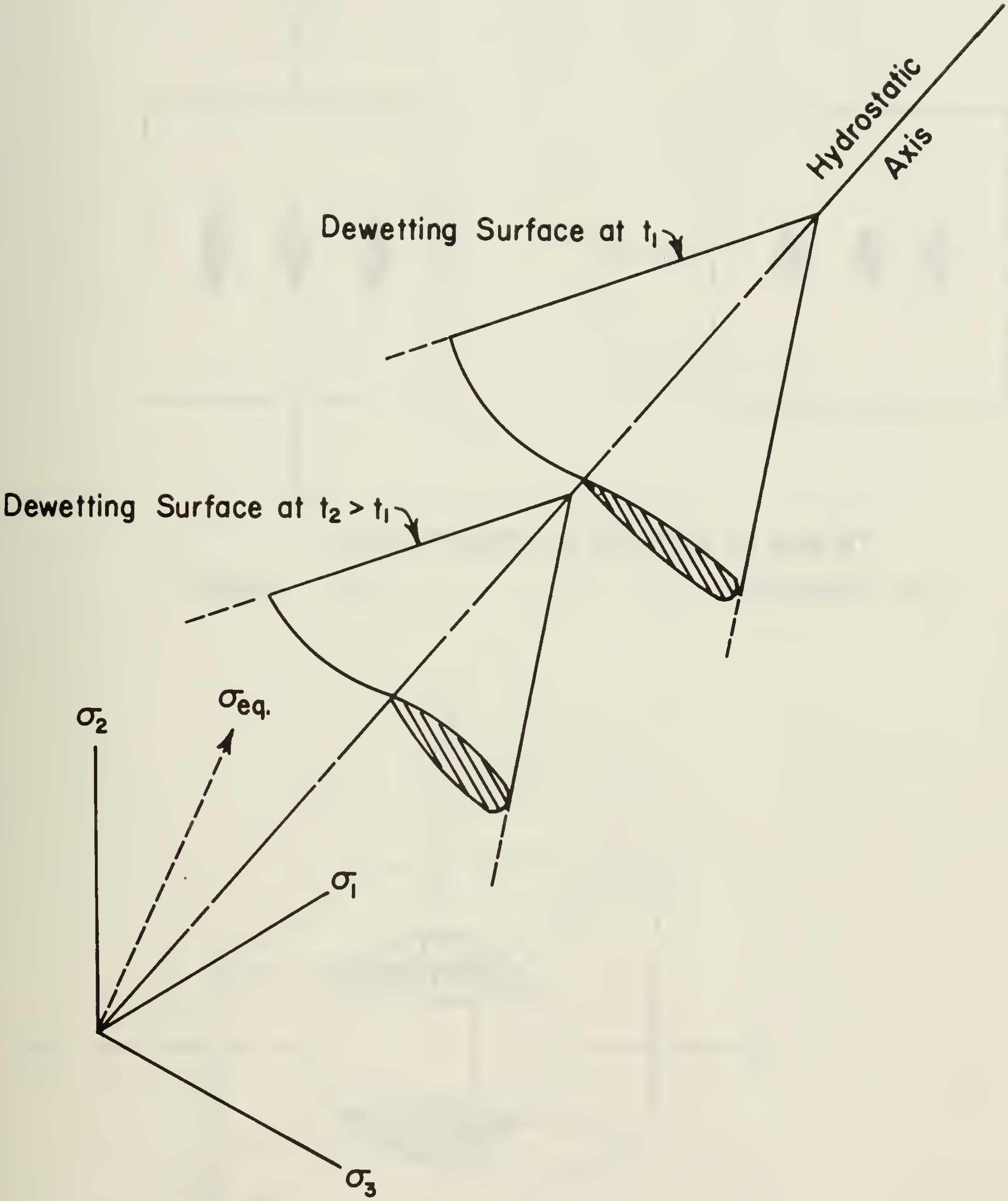


FIGURE 10. GENERALIZED DEWETTING CRITERION

ANISOTROPIC DEWETTED ELEMENT

FIGURE 11a FIGURE 11b

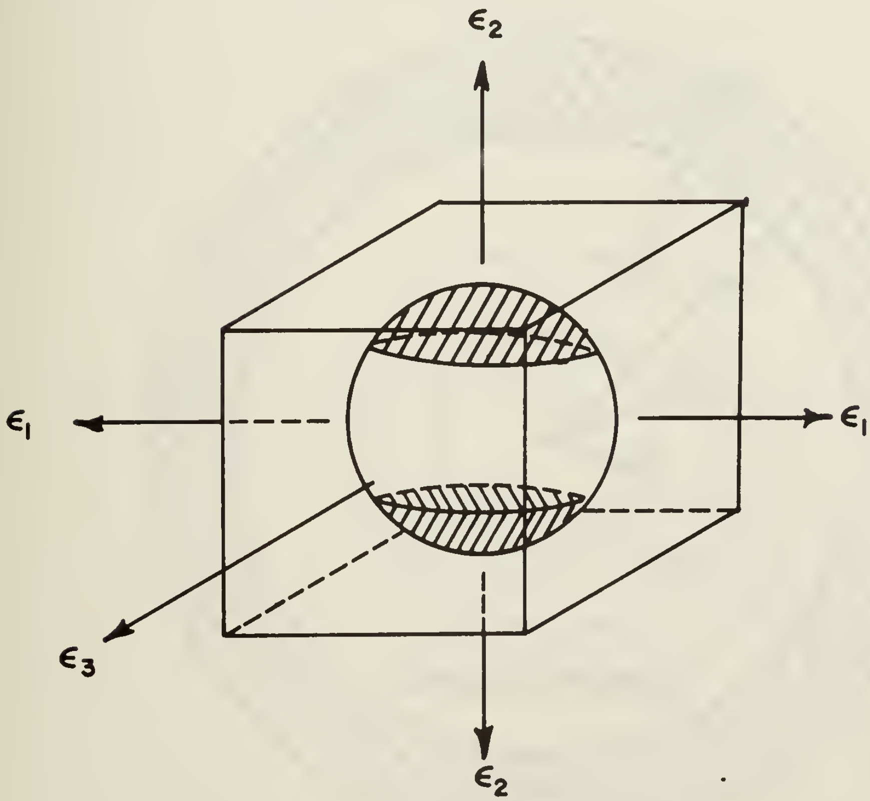


FIGURE 12. ELEMENT OF DEWETTED MATERIAL MODEL

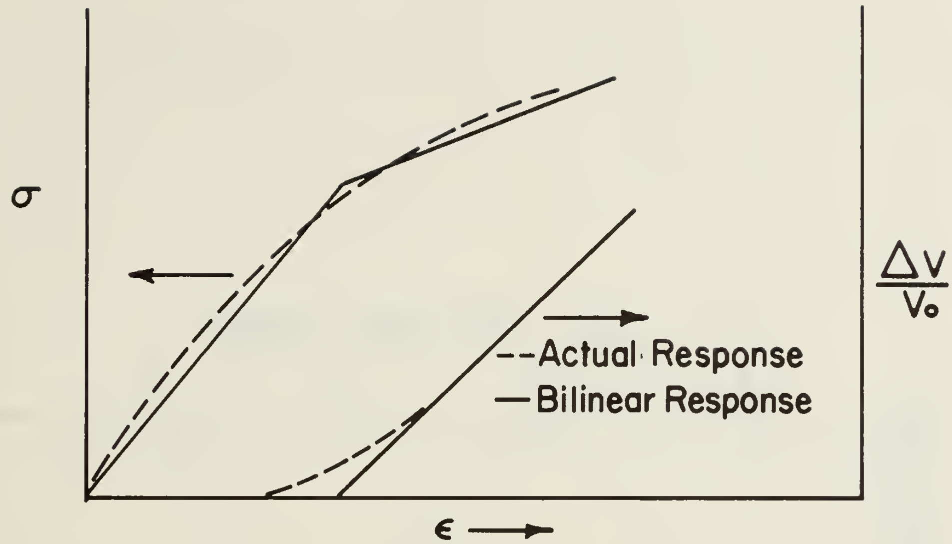


FIGURE 13. BILINEAR IDEALIZATION

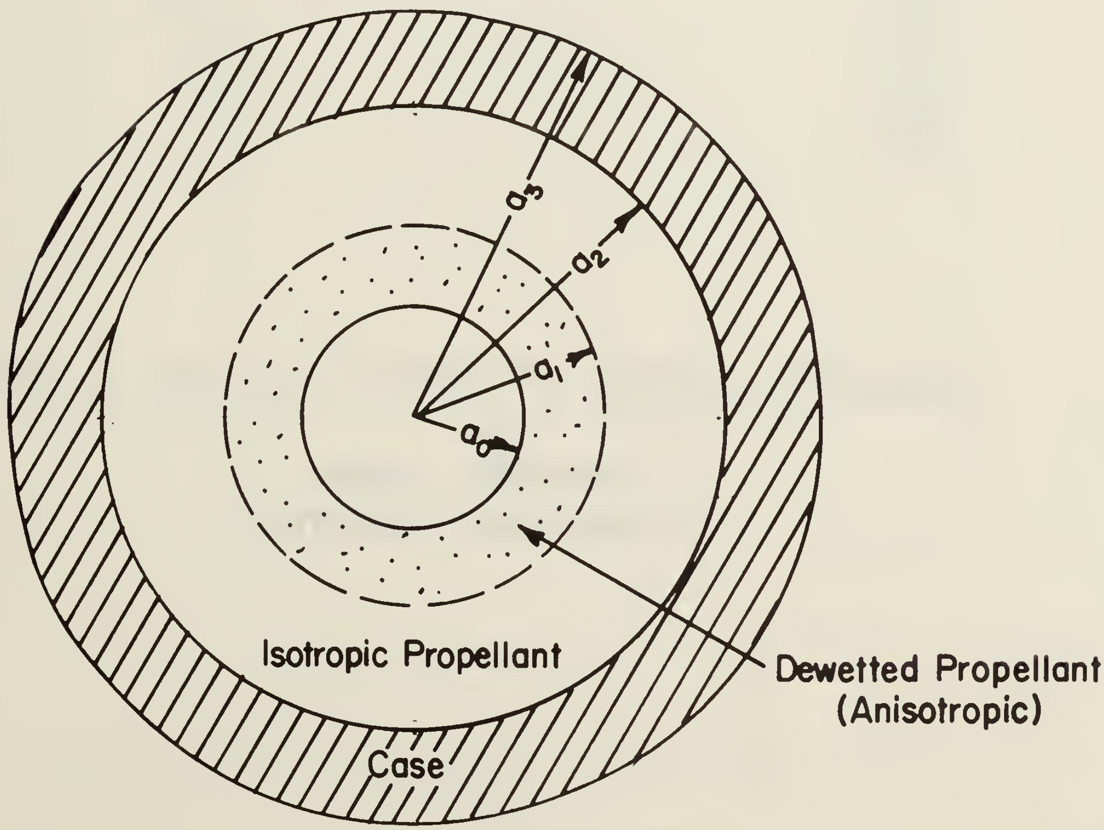


FIGURE 14. ROCKET MOTOR WITH DEWETTING

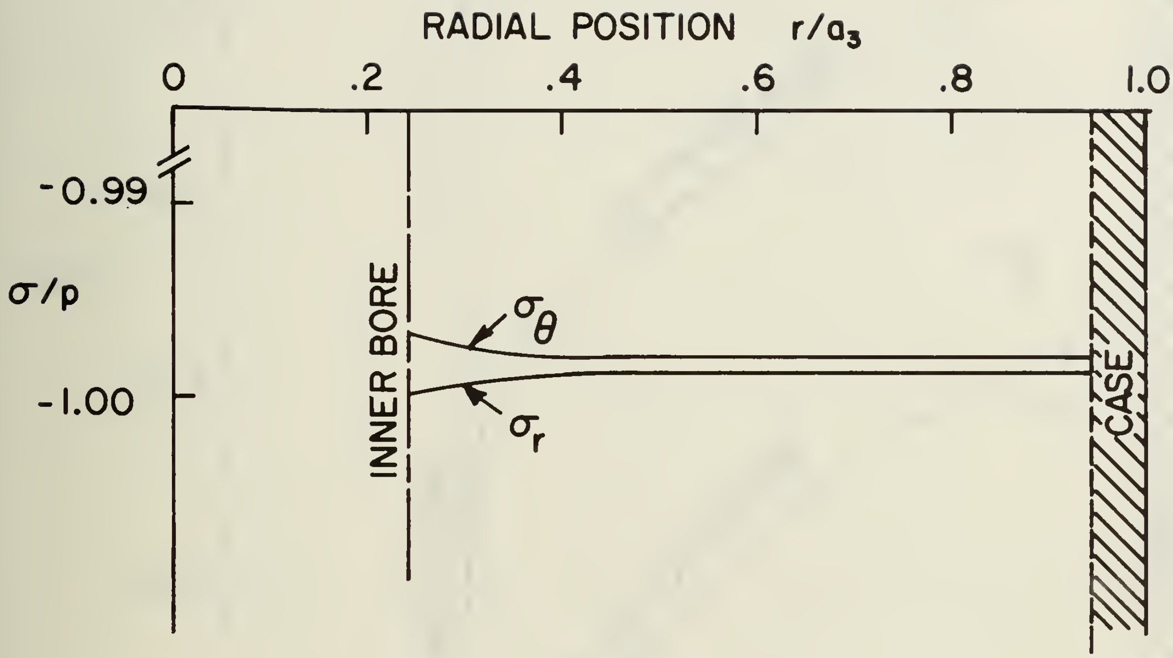


FIGURE 15. RATIO OF STRESS TO INTERNAL PRESSURE
VS
RADIAL POSITION
(ISOTROPIC SOLUTION)

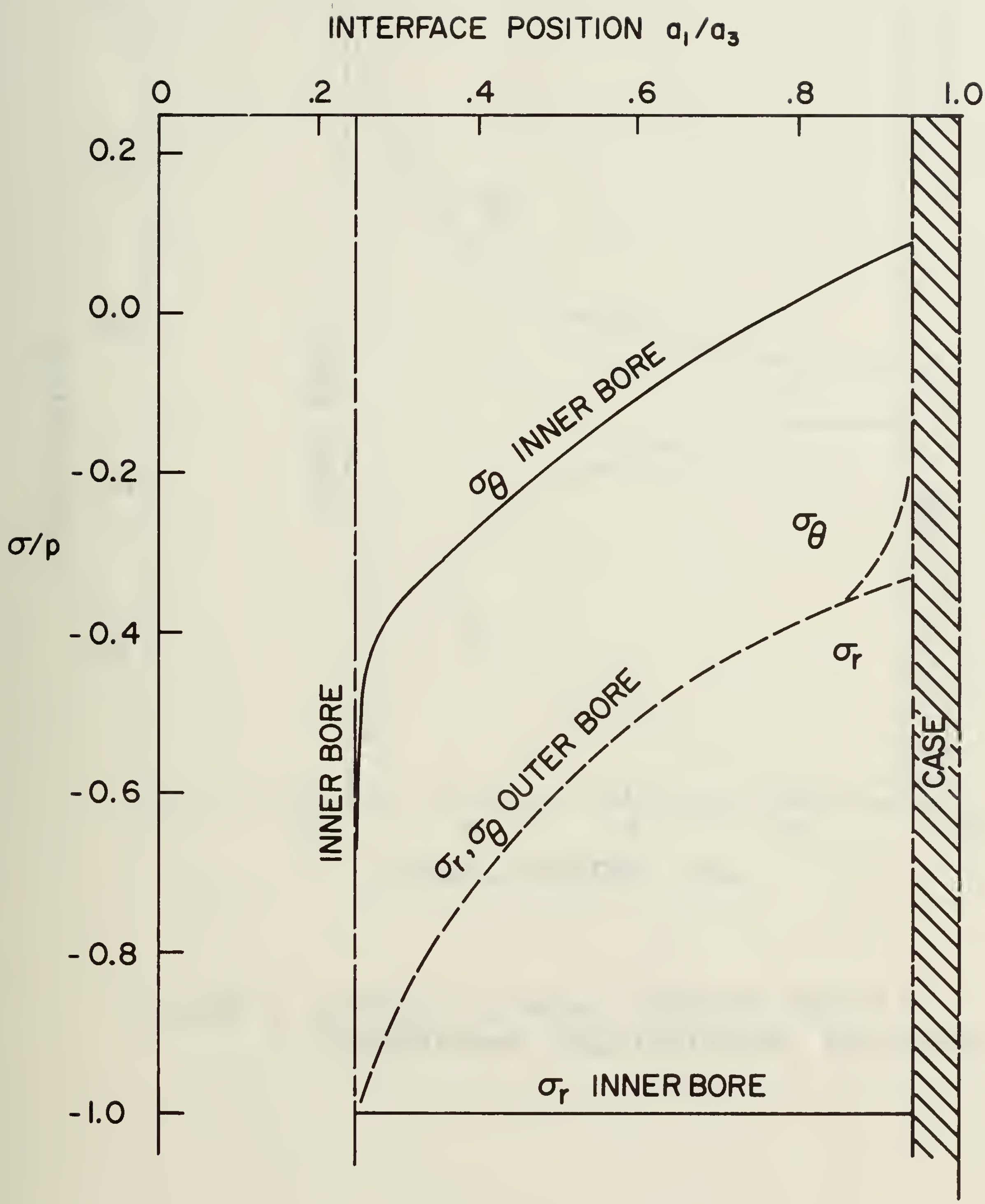


FIGURE 16. RATIO OF STRESS TO INTERNAL PRESSURE
VS
POSITION OF INTERFACE
(ANISOTROPIC SOLUTION)

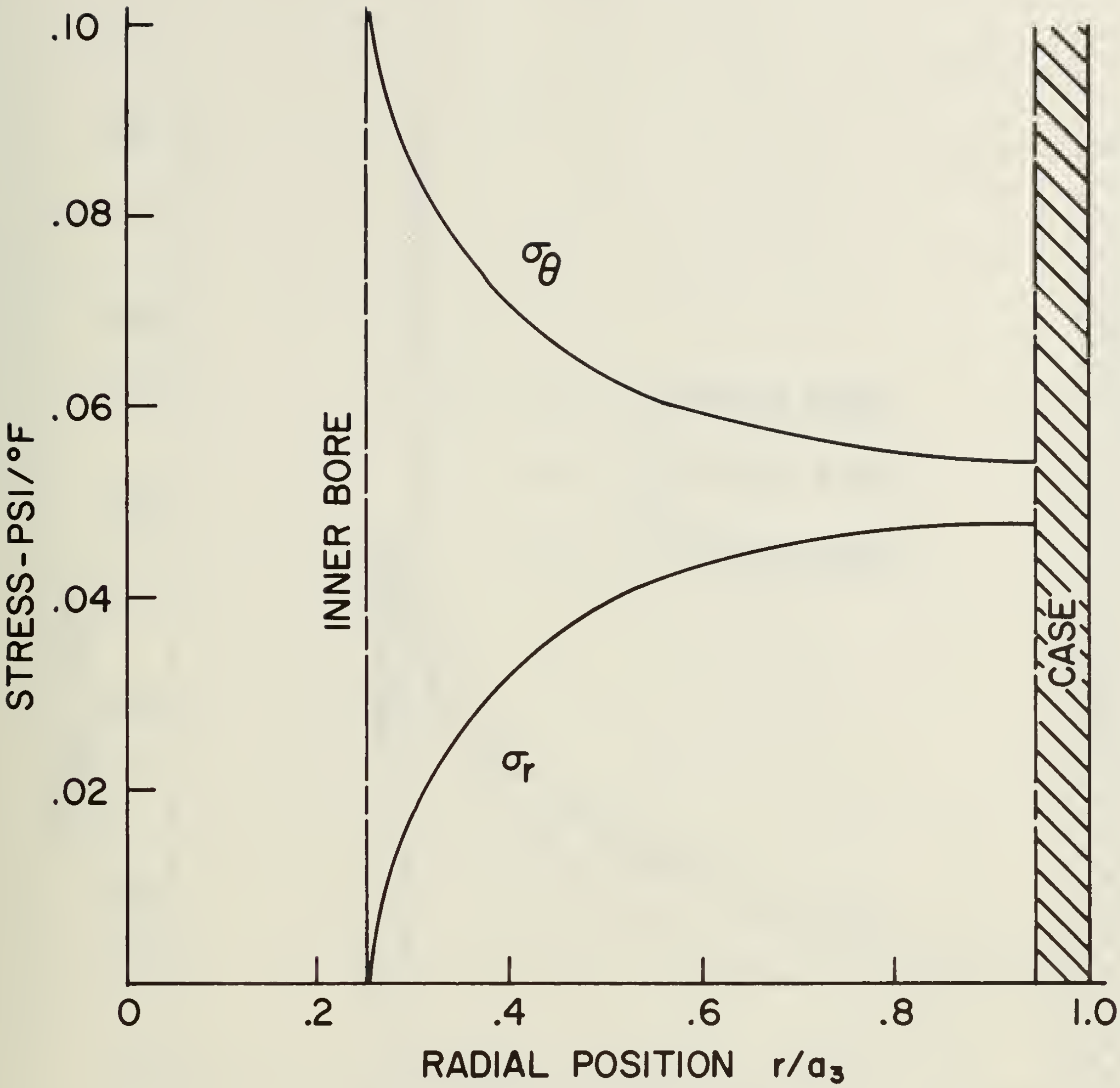


FIGURE 17. STRESS VS. RADIAL POSITION FOR 1° F TEMPERATURE DROP (ISOTROPIC SOLUTION)

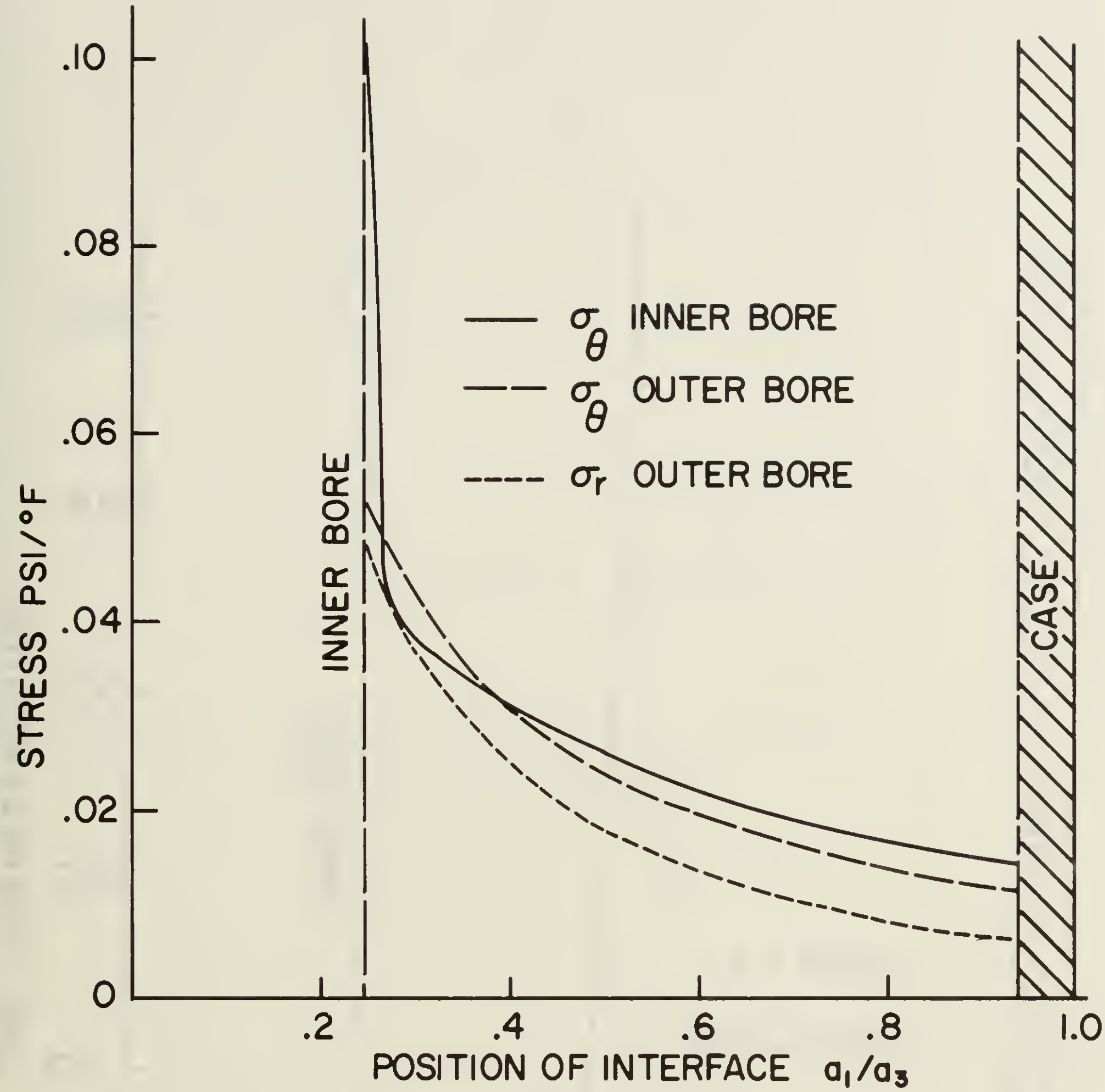
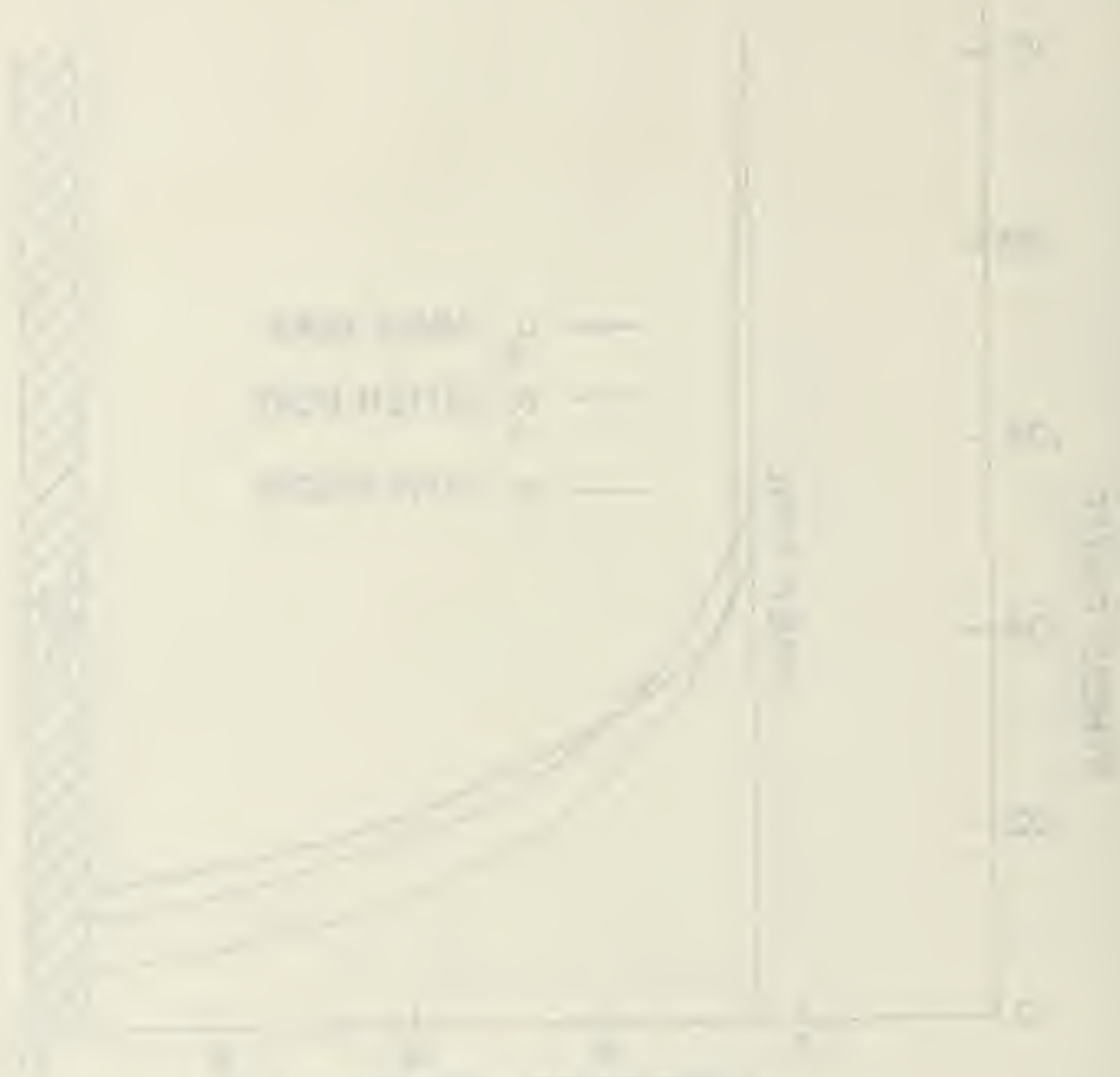


FIGURE 18. STRESS VS. INTERFACE POSITION FOR 1° F TEMPERATURE DROP (ANISOTROPIC SOLUTION)



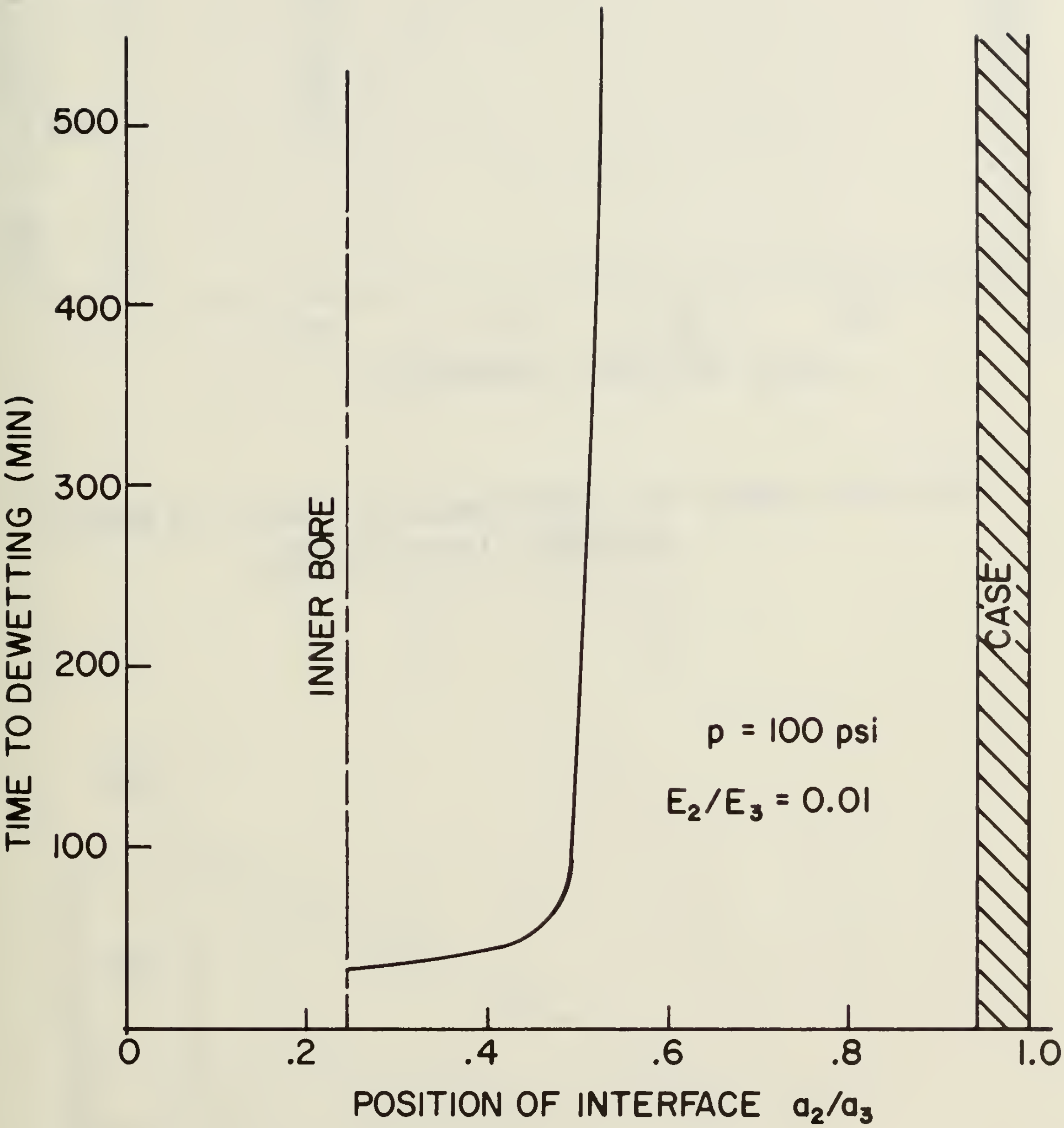


FIGURE 19. TIME TO DEWETTING VS. RADIAL POSITION (INTERNAL PRESSURIZATION)

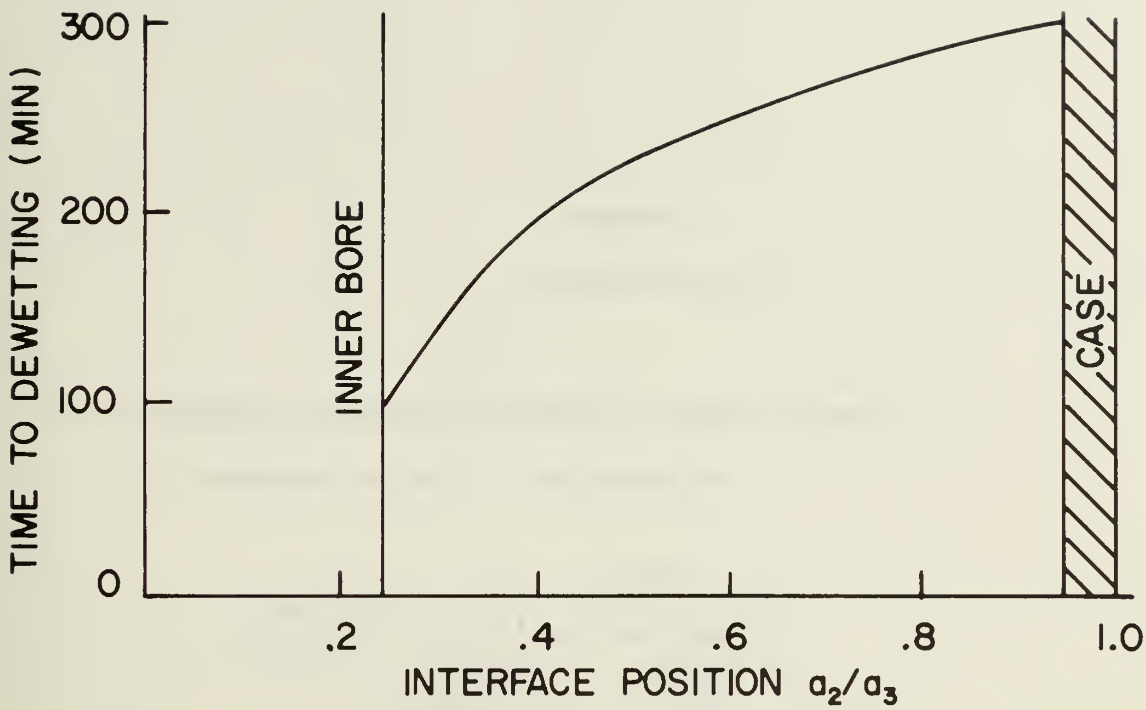


FIGURE 20. TIME TO DEWETTING VS. RADIAL POSITION (EQUILIBRIUM COOLING)

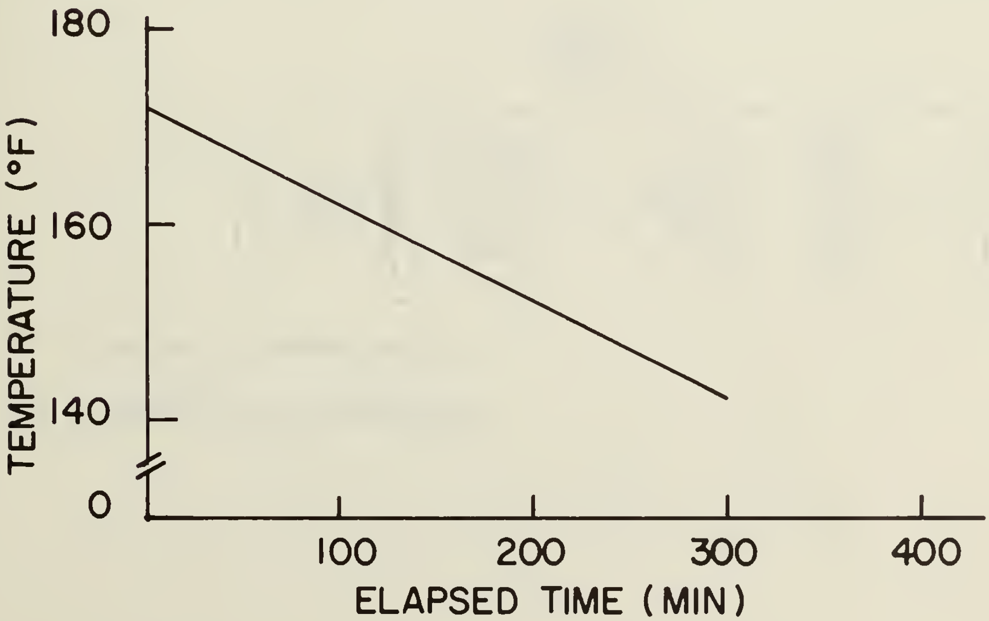


FIGURE 21. COOLING SCHEDULE FOR EQUILIBRIUM COOLING PROBLEM

APPENDIX I

STRESS FORMULAS

A. Internally Pressurized Cylinder - Plane Stress

1. Isotropic solution - no dewetting

$$\sigma_r = \left(\frac{a_o}{r}\right)^2 \left(\frac{q_2 - P}{1 - \left(\frac{a_o}{a_2}\right)^2}\right) + \frac{P\left(\frac{a_o}{a_2}\right)^2 - q_2}{1 - \left(\frac{a_o}{a_2}\right)^2}$$

$$\sigma_\theta = \frac{a_o^2}{r^2}\left(\frac{P - q_2}{1 - \left(\frac{a_o}{a_2}\right)^2}\right) + \frac{P\left(\frac{a_o}{a_2}\right)^2 - q_2}{1 - \left(\frac{a_o}{a_2}\right)^2}$$

$$q_2 = \frac{2P\left(\frac{a_o}{a_2}\right)^2}{\left[1 - \left(\frac{a_o}{a_2}\right)^2\right]\left[\frac{1 + \left(\frac{a_o}{a_2}\right)^2}{1 - \left(\frac{a_o}{a_2}\right)^2} - \nu_2 + \frac{E_2}{E_3}\left(\frac{1 + \left(\frac{a_2}{a_3}\right)^2}{1 - \left(\frac{a_2}{a_3}\right)^2} + \nu_3\right)\right]}$$

P = internal pressure at a_o.

q_2 = pressure at interface a_2.

2. Solution with dewetting. Region 1 is anisotropic .

a. $\underline{a_o < r < a_1}$ anisotropic region

$$\sigma_r)_1 = \frac{\left\{P\left(\frac{a_o}{a_1}\right)^{K+1} - q_1\right\}\left(\frac{a_1}{r}\right)^{(-K+1)}}{1 - \left(\frac{a_o}{a_1}\right)^{2K}} - \frac{\left\{P - q_1\left(\frac{a_o}{a_1}\right)^{K-1}\right\}\left(\frac{a_o}{a_1}\right)^{K+1}\left(\frac{a_1}{r}\right)^{(K+1)}}{1 - \left(\frac{a_o}{a_1}\right)^{2K}}$$

$$\sigma_\theta)_1 = \frac{K\left[P\left(\frac{a_o}{a_1}\right)^{K+1} - q_1\right]\left(\frac{a_1}{r}\right)^{(-K+1)}}{1 - \left(\frac{a_o}{a_1}\right)^{2K}} + \frac{\left[P\left(\frac{a_o}{a_1}\right)^{(K-1)}\right]K\left(\frac{a_o}{r}\right)^{(K+1)}}{1 - \left(\frac{a_o}{a_1}\right)^{2K}}$$

b. $\underline{a_1 < r < a_2}$ isotropic region

$$\sigma_r)_2 = q_1\left(\frac{a_1}{a_2}\right)^2\left[\frac{1 - \left(\frac{a_2}{r}\right)^2}{1 - \left(\frac{a_1}{a_2}\right)^2}\right] - q_2\left[\frac{1 - \left(\frac{a_1}{r}\right)^2}{1 - \left(\frac{a_1}{a_2}\right)^2}\right]$$

$$\sigma_\theta)_2 = q_1\left(\frac{a_1}{a_2}\right)^2\left[\frac{1 + \left(\frac{a_2}{r}\right)^2}{1 - \left(\frac{a_1}{a_2}\right)^2}\right] - q_2\left[\frac{1 + \left(\frac{a_1}{r}\right)^2}{1 - \left(\frac{a_1}{a_2}\right)^2}\right]$$

$$q_1 = \frac{2PKT_2\left[1 - \left(\frac{a_1}{a_2}\right)^2\right]\left(\frac{a_o}{a_1}\right)^{(K+1)}}{T_1T_2 - 4\left(\frac{a_1}{a_2}\right)^2\frac{E_\theta^{(1)}}{E_2}\left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 - \left(\frac{a_1}{a_2}\right)^2\right]}$$

$$q_2 = \frac{4PK\left(\frac{a_o}{a_1}\right)^{K+1}\left(\frac{a_1}{a_2}\right)^2\left[1 - \left(\frac{a_1}{a_2}\right)^{2K}\right]\left[1 - \left(\frac{a_2}{a_3}\right)^2\right]}{T_1T_2 - 4\left(\frac{a_1}{a_2}\right)^2\frac{E_\theta^{(1)}}{E_2}\left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 - \left(\frac{a_1}{a_2}\right)^2\right]}$$

$$T_1 = K\left[1 + \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 - \left(\frac{a_1}{a_2}\right)^2\right] + \frac{E_\theta^{(1)}}{E_2}\left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 + \left(\frac{a_1}{a_2}\right)^2\right]$$

$$- \left(\nu_{\theta r}^{(1)} - \frac{E_\theta^{(1)}}{E_2}\nu_2\right)\left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 - \left(\frac{a_1}{a_2}\right)^2\right]$$

$$T_2 = \left[1 + \left(\frac{a_1}{a_2}\right)^2\right]\left[1 - \left(\frac{a_2}{a_3}\right)^2\right] + \frac{E_2}{E_3}\left[1 - \left(\frac{a_1}{a_2}\right)^2\right]\left[1 + \left(\frac{a_2}{a_3}\right)^2\right]$$

$$- \left(\nu_2 - \frac{E_2}{E_3}\nu_3\right)\left[1 - \left(\frac{a_1}{a_2}\right)^2\right]\left[1 - \left(\frac{a_2}{a_3}\right)^2\right]$$

$$K = \sqrt{\frac{E_\theta^{(1)}}{E_r^{(1)}}}$$

q_1 and q_2 are the pressures acting at the interfaces a_1 and a_2 respectively.

B. Isothermal Cooling - Plane Stress

1. Isotropic solution - no dewetting

$$\sigma_r = \frac{(\alpha_2 - \alpha_3)(T_o - T)E_2\left[1 - \left(\frac{a_o}{r}\right)^2\right]}{1 + \left(\frac{a_o}{a_2}\right)^2 - \nu_2\left[1 - \left(\frac{a_o}{a_2}\right)^2\right] - \frac{E_2}{E_3}\left[\frac{1 - \left(\frac{a_o}{a_2}\right)^2}{1 - \left(\frac{a_3}{a_2}\right)^2}\right]}$$

$$\sigma_\theta = \frac{(\alpha_2 - \alpha_3)(T_o - T)\, E_2\left[1 + \left(\frac{a_o}{r}\right)^2\right]}{1 + \left(\frac{a_o}{a_2}\right)^2 - \nu_2\left[1 - \left(\frac{a_o}{a_2}\right)^2\right] - \frac{E_2}{E_3}\left[\frac{1 - \left(\frac{a_o}{a_2}\right)^2}{1 - \left(\frac{a_3}{a_2}\right)^2}\right]}$$

α_2, α_3 are the linear coefficients of thermal expansion in regions two and three respectively.

T_o is the stress-free temperature.

2. Anisotropic solution - region 1 has dewetted.

It is assumed that the coefficient of thermal expansion does not change with dewetting.

a. $a_o < r < a_1$

$$\sigma_r)_1 = 2F\left[\left(\frac{a_1}{r}\right)^{1-K} - \left(\frac{a_1}{r}\right)^{1+K}\left(\frac{a_o}{a_1}\right)^{2K}\right]$$

$$\sigma_\theta)_1 = 2KF\left[\left(\frac{a_1}{r}\right)^{1-K} + \left(\frac{a_1}{r}\right)^{1+K}\left(\frac{a_o}{a_1}\right)^{2K}\right]$$

b. $a_1 < r < a_2$ Isotropic region

$$\sigma_r = -2F\left[\frac{1 - \left(\frac{a_o}{a_1}\right)^{2K}}{1 - \left(\frac{a_1}{a_2}\right)^2}\right]\left[\left(\frac{a_1}{a_2}\right)^2 - \left(\frac{a_1}{r}\right)^2\right] + FT_1\left[\frac{1 - \left(\frac{a_1}{r}\right)^2}{1 - \left(\frac{a_1}{a_2}\right)^2}\right]$$

$$\sigma_\theta = -2F\left[\frac{1 - \left(\frac{a_o}{a_1}\right)^{2K}}{1 - \left(\frac{a_1}{a_2}\right)^2}\right]\left[\left(\frac{a_1}{a_2}\right)^2 + \left(\frac{a_1}{r}\right)^2\right] + FT_1\left[\frac{1 + \left(\frac{a_1}{r}\right)^2}{1 - \left(\frac{a_1}{a_2}\right)^2}\right]$$

$$F = \frac{(\alpha_2 - \alpha_3)(T_o - T)E_2}{T_2 + \frac{E_2}{E_3} T_1 \left[\frac{1 + \left(\frac{a_2}{a_3}\right)^2}{1 - \left(\frac{a_2}{a_3}\right)^2} + \nu_3 \right]}$$

$$T_1 = \left[1 + \left(\frac{a_1}{a_2}\right)^2\right]\left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right] + \nu_2 \left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 - \left(\frac{a_1}{a_2}\right)^2\right]$$

$$+ \frac{E_2}{E_\theta^{(1)}} \left\{ K \left[1 + \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 - \left(\frac{a_1}{a_2}\right)^2\right] - \nu_{\theta r}^{(1)} \left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 - \left(\frac{a_1}{a_2}\right)^2\right] \right\}$$

$$T_2 = \left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right]\left[1 - \left(\frac{a_1}{a_2}\right)^2\right](1 - \nu_2^2) + \left\{ 1 + \left(\frac{a_1}{a_2}\right)^2 \right.$$

$$\left. - \nu_2 \left[1 - \left(\frac{a_1}{a_2}\right)^2\right] \right\} \left\{ K \left[1 + \left(\frac{a_o}{a_1}\right)^{2K}\right] - \nu_{\theta r}^{(1)} \left[1 - \left(\frac{a_o}{a_1}\right)^{2K}\right] \right\} \frac{E_2}{E_\theta^{(1)}}$$

C. To apply the above formulas to <u>plane strain</u> conditions, replace modulus values with the primed quantities listed below. Superscripts refer to regions of cylinder.

$$E_r^{(1)'} = \frac{E_r^{(1)}}{1 - \frac{E_z^{(1)}}{E_r^{(1)}} \nu_{rz}^{2(1)}}$$

$$E_\theta^{(1)'} = \frac{E_\theta^{(1)}}{1 - \frac{E_z^{(1)}}{E_\theta^{(1)}} \nu_{\theta z}^{2(1)}}$$

$$\nu_{\theta r}^{(1)'} = \frac{\nu_{\theta r}^{(1)} + \frac{E_z}{E_r} \nu_{rz}^{(1)} \nu_{\theta z}^{(1)}}{1 - \frac{E_z^{(1)}}{E_\theta^{(1)}} \nu_{\theta z}^{2(1)}}$$

$$K' = \sqrt{\frac{E_\theta^{(1)'}}{E_r^{(1)'}}}$$

$$E_2' = \frac{E_2}{1 - \nu_2^2} \qquad E_3' = \frac{E_3}{1 - \nu_3^2}$$

$$\nu_2' = \frac{\nu_2}{1 - \nu_2} \qquad \nu_3' = \frac{\nu_3}{1 - \nu_3^2}$$

APPENDIX II

EVALUATION OF STRESS FORMULAS

Estimation of Material Properties

For sample calculations of stresses within a hollow encased cylinder, the following representative values of material properties were selected.

For the undewetted isotropic propellant: $E_2 = 300$ psi

$\nu_2 = 0.5$

$\alpha_2 = 2.21 \times 10^{-5}$ in./in./°F

For the case (steel): $E_3 = 3 \times 10^7$ psi

$\nu_3 = .33$

$\alpha_3 = 6.5 \times 10^{-6}$ in./in./°F

In order to obtain estimates of material properties of the propellant after dewetting, the effect of vacuole formation is considered. Since during dewetting the vacuoles form and extend in the direction of the maximum tensile stress, which in the cases under consideration is the tangential direction, loss of reinforcement is most severe in this direction. In radial and tangential directions, the binder-filler bond is still intact, consequently modulus values for these directions will be less affected by dewetting. As a first approximation it is assumed that the modulus values E_r and E_z will not change during dewetting, and will remain equal to the isotropic modulus E_2. With this assumption the Poisson's ratios ν_{zr} and ν_{rz} will also remain at their isotropic value ν_2.

To estimate the extent to which tangential modulus is changed with dewetting, uniaxial tensile test results are considered. Francis and Carlton[20] report for a composite propellant investigated by them, an approximately 50% reduction in uniaxial Poisson's ratio was associated with the dewetted state. For the purposes of the present calculation, the same reduction is assumed for the corresponding ratio $\nu_{\theta r} = \nu_2/2$. Similarly $\nu_{\theta z} = \nu_2/2$. With these values determined, the rest of the desired moduli may be obtained from requirements of symmetry in the modulus matrix. In summary, the following values are chosen for anisotropic properties of the dewetted material.

$$E_r^{(1)} = E_z^{(1)} = E_2$$

$$E_\theta^{(1)} = E_{2/2}$$

$$\nu_{rz}^{(1)} = \nu_{zr}^{(1)} = \nu_2$$

$$\nu_{\theta r} = \nu_{\theta z} = \nu_{2/2}$$

For the plane strain solution (see appendix I) the moduli are replaced with primed values given by

$$E_r' = \frac{E_r}{1 - \nu_{rz}^2} = \frac{E_2}{1 - \nu_2^2}$$

$$E_\theta' = \frac{E_\theta}{1 - \frac{E_r}{E_\theta}\nu_{\theta z}^2} = \frac{E_2}{2\left(1 - \frac{\nu_2^2}{2}\right)}$$

$$\nu_{\theta r}' = \frac{\nu_{\theta r} + \nu_{rz}\nu_{\theta z}}{1 - \frac{E_r}{E_\theta}\nu_{\theta z}^2} = \frac{\nu_2(1 + \nu_2)}{2\left(1 - \frac{\nu_2^2}{2}\right)}$$

In the absence of any information to the contrary, it is assumed that the coefficient of thermal expansion remains isotropic and unchanged from the value of the undewetted material.

Geometry

A particular set of radius ratios was chosen corresponding to the dimensions of an analogue motor for which some test data are available. These ratios are

$$\frac{a_o}{a_2} = 0.2465$$

$$\frac{a_2}{a_3} = 0.934$$

Results

A computer program was written to calculate the stresses developed at various points within the propellant as a function of the position of the interface between anisotropic and isotropic propellant per pound of internal pressure. Results are shown in Figures 15 and 16. With the high modulus steel liner, the isotropic solution is very nearly uniform hydrostatic pressure, reflecting the choice of an incompressible Poisson's ratio. For the values of anisotropic material properties chosen, the effect of dewetting is to decrease stress levels throughout the propellant grain. For the high modulus value of the case used in this calculation, all stresses within the propellant are compressive, whether dewetted or not.

A second program calculates the stresses developed at various points within the propellant as a function of the interface between anisotropic and isotropic material per degree Fahrenheit cooling below the stress-free

temperature of the motor. For this case all stresses are tensile, and again a decrease in both tangential and radial stresses occurs throughout the propellant as the anisotropic layer grows. Results are given in Figures 17 and 18.

Each of these programs also calculates an equivalent stress for use in the reaction rate equation. The particular form chosen to describe equivalent stress is discussed in appendix III.

APPENDIX III

CALCULATION OF POSITION OF DEWETTING INTERFACE WITH TIME

An equivalent uniaxial stress function is chosen in agreement with Lindsey's elastic dewetting criterion. From equation (6),

$$\sigma_{eq} = \frac{\sigma_u}{\sigma_t}\frac{I_1}{3} + \frac{\sqrt{3}}{3}\left(3 - \frac{\sigma_u}{\sigma_t}\right)J_2^{1/2}$$

Lindsey[5] has reported that for composite propellants, the dewetting stress in triaxial tension σ_t has been found to be approximately twice the dewetting stress in uniaxial tension σ_u. With this information,

$$\sigma_{eq} = \frac{I_1}{6} + \frac{5\sqrt{3}}{6} J_2^{1/2}$$

For plane strain conditions:

$$I_1 = \sigma_\theta + \sigma_r \qquad\qquad J_2^{1/2} = \frac{|\sigma_\theta - \sigma_r|}{2}$$

For the cases under consideration, $\sigma_\theta > \sigma_r$, thus the absolute value signs may be removed to yield $\sigma_{eq} = .97\sigma_\theta - .47\sigma_r$. This relation was used during computation.

Parameters chosen for the reaction rate equation are those obtained by Graham and Robinson[12] for rupture of filled PBAN propellant:

$A = -18.1 \qquad\qquad B = 10104 \qquad\qquad C = 0.055$

While these values cannot be assumed to be typical of dewetting in PBAN propellant or applicable to the entire temperature range, they will be satisfactory for sample calculations.

For the internal pressurization of a cylinder in steel casing, stresses obtained from the elastic solution produce negative values for the equivalent stress throughout the cylinder, indicating that dewetting is entirely suppressed. For the purposes of calculation, a much more flexible case was assumed, with a ratio of moduli

$$\frac{E_{propellant}}{E_{case}} = 0.01$$

The time to dewetting at each radial position is calculated with the above choice of constants and the geometry described in appendix II. Results are given in Figure 19.

For the thermal cool-down problem, a stress-free temperature of the encased propellant was required. This was chosen to be 172°F. A rate of cooling of 0.1°F per minute was chosed for calculation purposes. Figures 20 and 21 show the calculated time to dewetting at each radial position as the cylinder and steel case are cooled isothermally. An interesting result in this case is that the entire cylinder has dewetted by the time the temperature has dropped thirty degrees from the stress-free temperature.

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13. ABSTRACT

Fundamental concepts have been formulated for the mechanical behavior of isotropic, elastic materials that dewet. This has been accomplished through an examination of the stability concepts underlying the classical inviscid theory of plasticity. From a phenomenological viewpoint, the two theoris differ in the nature of unloading, the dilatation behavior, the form taken by the dewetting criterion, which is analogous to the yield criterion of plasticity theory. Some restrictions to an allowable functional form of the dewetting criterion are developed, and a specific criterion compatible with these restrictions is suggested. A time-temperature superposition method is developed, using the proposed criterion coupled with reaction rate theory. Stress analysis for materials which exhibit anisotropy after dewetting is discussed, and sample problem solutions are given for quasi-elastic pressurization and slow cool-down of an encased hollow cylinder in plane strain, in which the extent of dewetting increases with time.

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